Introduction to Trigonometric Functions Using Triangles - Polished Transcript

0:01

Welcome to a video that introduces the trigonometric functions in terms of right triangles. If you need to build a ramp with an incline of 6 degrees and it must reach a height of 2 feet, how long would the ramp have to be? We can solve this type of problem using trigonometry.

0:21

There are trigonometric functions: sine, cosine, tangent, cosecant, secant, and cotangent. Customarily, they are abbreviated using the notation shown. These trig functions can be defined in terms of particular ratios of sides of right triangles. Here are the sine, cosine, and tangent functions in terms of ratios of sides of right triangles.

0:52

If we start at angle A, sine is the ratio of the opposite side over the hypotenuse. Remember, the hypotenuse is always opposite the right angle. If you were to bisect the right angle, the side you run into would always be the hypotenuse. The opposite and adjacent sides change depending on which angle you're referring to.

1:19

For angle A, if we bisect it, the side we run into is the opposite side. So the sine of A is the ratio of the opposite over the hypotenuse, or A/C.

1:42

The cosine of angle A is the ratio of the adjacent side over the hypotenuse. If this is the opposite side from angle A, then this is the adjacent side, because this is always the hypotenuse. So cosine of A is B/C.

2:05

Tangent of angle A is the ratio of the opposite side over the adjacent side, or A/B.

2:15

Now looking at the second row, we refer to angle B. This changes the orientation of the opposite and adjacent sides. If we bisect angle B, the opposite side is now B, and the adjacent side is A. Regardless of which angle we refer to, the ratios remain the same: sine is opposite over hypotenuse, cosine is adjacent over hypotenuse, and tangent is opposite over adjacent.

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2:55

For angle B, sine is B/C, cosine is A/C, and tangent is B/A.

3:13

One way to remember these is the acronym SOH-CAH-TOA: sine is opposite over hypotenuse, cosine is adjacent over hypotenuse, and tangent is opposite over adjacent.

3:29

All right triangles with the same acute angles are similar. If triangles are similar, the trigonometric functions produce the same ratios because the triangles are proportional. Looking at the large right triangle and the small green triangle, they share a common angle and both have right angles, so they are similar.

4:00

Labeling the full lengths: if the hypotenuse is 10 and the opposite side is 6, then sine A is 6/10. In the smaller triangle, opposite over hypotenuse is 3/5. These are equivalent ratios.

4:26

The same applies to cosine A: 8/10 in the large triangle and 4/5 in the small triangle. Tangent A is 6/8 in the large triangle and 3/4 in the small triangle-again equivalent.

4:51

Now we define the other three trigonometric functions: cosecant, secant, and cotangent. These are reciprocals of sine, cosine, and tangent. Cosecant A is hypotenuse over opposite. Secant A is hypotenuse over adjacent. Cotangent A is adjacent over opposite.

5:33

For angle A, the hypotenuse is C, the opposite is A, and the adjacent is B. So cosecant A is C/A, secant A is C/B, and cotangent A is B/A.

6:00

Let's determine the sine, cosine, and tangent of angle A. Identify the three sides: the hypotenuse, the opposite side, and the adjacent side. Before finding the values, we notice the hypotenuse length is missing, so we apply the Pythagorean Theorem: C squared equals 5 squared plus 12 squared, which is 25 plus 144, giving 169. So C equals 13. This is a 5-12-13 right triangle.

7:00

Sine A is 12/13. Cosine A is 5/13. Tangent A is 12/5.

7:24

Let's try one more. Again, we are missing the length of the third side. Using the Pythagorean Theorem with hypotenuse 8 and one leg 4, we find the missing side is the square root of 48, which is four times the square root of three.

8:05

We are asked for cosecant, secant, and cotangent of angle B. For angle B, the opposite side is 4 and the adjacent side is four times the square root of three. Cosecant is the reciprocal of sine, so hypotenuse over opposite: 8/4 equals 2. Secant is the reciprocal of cosine: hypotenuse over adjacent, 8 divided by four times the square root of three, which simplifies to 2 divided by the square root of three. When rationalized, this becomes two times the square root of three over three. Cotangent is the reciprocal of tangent: adjacent over opposite, four times the square root of three divided by 4, which simplifies to the square root of three.

9:26

I hope you found this video helpful. Have a good day.