Transcript for A 2.4 Centroids and Center of Mass via Composite Parts

0:01

Hello. In today's video lecture, we're going to talk about finding centroids and the center of mass using composite parts - an alternate method for determining centroid locations. So far, we've focused on finding the centroid or center of mass through integration. However, performing these calculations by hand can become complex and difficult to manage as shapes become more complicated.

For example, consider the arch-shaped figure shown. To find its centroid through integration, we would need mathematical functions describing its width and height as we move left-right and up-down. While possible, these equations can become quite complex, and integrating them only increases the difficulty.

Because of this, we often use alternate methods. One solution is the method of composite parts, which takes advantage of previously computed integrals for common shapes. Another solution is to use software tools - most CAD programs can compute centroid or center-of-mass values automatically. We will not cover software tools here. Instead, we will focus on the method of composite parts as an alternative to integration.

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Finding the centroid using the method of composite parts works by breaking down a complex shape into simple shapes. Each simple shape has a known centroid location, typically listed in a centroid table. These table values were originally computed using integration, but they are expressed in terms of general variables such as width, height, or radius.

For each simple shape used in the composite method, we need two things: its area and its centroid coordinates, X and Y.

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Centroid tables provide generalized centroid locations for many common shapes. These values must be adapted to the specific dimensions of the shape in your problem.

For example, consider a triangle whose centroid we want to locate. The centroid table shows a triangle with base B and height H. The centroid is located at B/3 from the right angle along the base and H/3 upward from the base.

If our triangle is six inches wide and four inches tall, then H/3 = 4/3 inches gives the Y-coordinate. For the X-coordinate, B/3 = 6/3 = 2 inches measured from the right angle. Because the triangle in our problem is mirrored relative to the table figure, this distance is measured from the opposite side. Thus, the centroid is located four inches from the left edge and four-thirds of an inch upward.

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Now let's apply the method of composite parts more broadly. Suppose we want to find the centroid of the area shown. Step one is to break the shape into simpler parts. Each part must match a shape listed in the centroid table. For example, if the shape includes a hexagon but the table does not, we cannot use the hexagon as a simple part.

Regular areas count as positive areas, while holes or cutouts count as negative areas. In our example, we divide the shape into three parts: a semicircle (shape 1), a rectangle (shape 2), and a triangular cutout (shape 3). The triangular cutout is treated as a negative area.

After dividing the shape, we number the parts and create a table. For each part, we list its area and its centroid coordinates X and Y relative to a single origin. These are not the raw table values - they must be shifted to match the chosen coordinate system.

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For the semicircle, the centroid table shows the centroid located at a distance of 4/(3pi)*R from the flat side, measured along the axis of symmetry. Because the table's figure is rotated relative to our shape, we mentally rotate it and then determine the centroid location relative to our chosen origin.

For the rectangle, the centroid is simply at half the width and half the height. To locate it relative to the origin, we add the radius of the semicircle plus half the rectangle's width.

Once all X and Y values are determined, we compute the overall centroid. The total X is the sum of (area * X) for each part, divided by the total area. The same process applies for Y.

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For three-dimensional shapes, the process is the same except we use volumes instead of areas, and we compute X, Y, and Z. For example, consider a cone on top of a cylinder. We compute the volume of each shape and find X, Y, and Z for each. Then we calculate the overall centroid using the same weighted-average method.

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The center of mass calculation follows the same structure as the volume-based centroid calculation, except we use mass instead of volume. For example, if a brass cone sits on top of an aluminum cylinder, we compute the mass of each part and use mass-weighted averages to find the overall center of mass.

10:10

These are the procedures for finding centroids in 2D, centroids in 3D, and centers of mass using composite parts. Thank you for watching.