Centroids via the Method of Composite Parts - Adaptive Map Worked Example 1

(0:00 - 2:30)

Hello, for this problem, we've got a complex shape, so we're asked to find the x and y coordinates of the centroid of the shape shown below. So, we've got kind of a rectangular section with a circular cutout, and there's this triangular section over here. So, we're going to use the method of composite parts to find the centroid, the x and y coordinates of those in terms of our coordinate system.

So, I've defined (it's an important part of each problem), the y-axis and the x-axis, making this point down here in the corner our origin point. So, for each of this, each of these, we need to come up with a table. So, we're going to need to break this down into parts, and right up front, let's break it down into three parts.

We're going to create part one is going to be this rectangle here, it's four inches wide by three inches tall. Part two is going to be the circular cutout, so I've got a circular hole, it's two inches in diameter, it's two inches this way, 1.5 inches this way, that's the center. So, it's got the same center as the rectangular point there, or the rectangular area there.

And then area three is this triangular area over here, so I know that based on this being four, this being eight, this is going to be four inches wide and then three inches tall. So, I need to create a table and I'm going to find for each of these three shapes, I'm going to find the area, I'm going to find the x-coordinate for the centroid of that shape, and I'm going to find the y-coordinate for the centroid of that shape. So, let's go ahead and create our table.

So, I need the shape, the area, the x-coordinate for the centroid, and the y-coordinate for the centroid. So, I've got shape one, two, and three. And creating this table is kind of the centerpiece of any of the any time we are going to use the method of composite parts.

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So, for shape one, let's go back to our problem. We've got a rectangular area. So, for our rectangular area we've got, it's four inches wide by three inches tall, our area is going to be 12 square inches. So, we end up with 12 inches squared right there.

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All right, next up is my circle. So, this is a cutout, that means I'm going to count it as a negative area. So, it's going to have a negative area and the area of that circle is going to be pi r squared, diameter is two inches, my radius is one inch, so pi times one squared, that gives me roughly 3.14 square inches, and again it's going to be negative because it's a cutout.

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So, finally, area three, let's look at the area of that. It's a triangle, so one-half base times height, the base from here to here, that's going to be four inches, and then the height is going to be three inches. So, for area three, one-half base times height gives me six square inches there.

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All right, next up, I need to find the centroid for each of these shapes. So, for this, I'm going to go to the tables, and online you can see the link to the tables there, but I've written down some of the important ones here. So, taking from that table, I've got three shapes, I've got my rectangle, my right triangle, and my circle, and each one of these I've got some information on where the centroid is.

So, the first one is the rectangle, the centroid, the geometric center of this shape is going to be half of the way across and half of the way up, that kind of makes sense. So, my base is going to be four inches in my diagram over there, so one half of my base is going to be two inches, and my height is going to be three inches on the diagram over here, so my height, one half of my height is going to be 1.5 inches, and both of these are measured from, I've got my origin point here, so I go over two inches, up 1.5 inches, and that's the center of my rectangle. So, easy enough, x bar is going to be two inches, and y bar is going to be 1.5 inches.

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All right, so next we have the circle, and I really didn't draw in any dimensions for this circle, because usually we measure everything with respect to the center of the circle, and the center of the circle is the centroid of a circle. So, I just need to locate the center of this circle over here, and so going from my origin point right here, the center of the circle, I know it's just kind of simply defined on the graph, it's two inches this way and 1.5 inches up. So, I end up with the same values here, the center of the circle is two inches over and 1.5 inches up.

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All right, finally I need to find the centroid of my triangle. So, for my triangle, the centroid is going to be somewhere over here, and looking at the table, I have this. So, I know that from the right angle, like right there, if I go one third of the way, the length of the base from that side, one third of the height of triangle up, I'm going to end up at the centroid.

So, if I look over here at shape three, I know I want to go from the right angle, which is going to be right here, I go one third of the base, so the base is four inches, so one third of that is going to be four thirds, so I need to go over four thirds, but I'm also defining everything from over here. So, I need to go, before I even get to the triangle, I need to go four inches over and an extra four thirds this way. So, four inches plus four thirds inches is going to give me a value of 5.33 inches.

That means the x coordinate for the centroid of this triangle is 5.33 inches over somewhere around here or so. All right, so now I need to look at the height of my centroid, and so going from the right-angle side, if I go one third of the height up, I end up at the centroid. So, looking here, if I go going from the right triangle side, if I go one third of the way up towards the point, I'm going to end up at my centroid.

So, one third of three inches, which is the height, is going to give me one inch, which is the height of my centroid there. All right, so once I filled in this graph, or once I fill in this table, this is most of the work that I need to do. Now I just need to perform the calculations.

So, x bar, we call this x bar t, it's the x coordinate for the overall centroid of the shape, is going to be equal to the sum of area times x bars over the sum of the areas. So, this is going to be area times x bar 1, so 12 times 2, plus, I can go to the next line, negative 3.14 times 2, plus, I go to the third line, 6 times 5.33. And divide that whole thing by the overall area. So, 12 minus 3.14 plus 6. All right, now I've got all the numbers plugged in, and if I solve this equation, I should end up with a value of 3.34 inches.

All right, next, we want to go ahead and do the y values. So, y bar t, this is the y coordinate, is going to be the area, area 1 times y bar 1, plus area 2 times y bar 2, plus area 3 times y bar 3. So, 12 times 1.5, plus negative 3.14 times 1.5, plus 6 times 1. And I could, again, I'm going to divide that by the total area, so 12 minus 3.14 plus 6. And if I do all this math, I should end up with about 1.29 inches. Over here, this is my x bar, and this is my y bar total.

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Um, so if I go back to my diagram, my x bar total, I'd have to go over about 3.34 inches, somewhere about this far. And I'd have to go up 1.29 inches, so not quite halfway. My centroid is going to be somewhere there.

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And again, I'm going over 3.34 and up 1.29. So, with that, we've found our centroid, and we've solved our problem. Thank you for watching, and I hope to see you again.