Transcript for 5.7 WP 004 - Analysis of Frames and Machines - JPM

0:02

Hello. For today's worked example, we're going to look at this problem. We have a TV tray with a maximum rated load of 100 pounds, and we're going to determine the forces acting on each of the legs in the setup. We assume the entire load is concentrated on one side of the tray, where the set of legs shown is supporting the full 100-pound load. We're also assuming the load is centered left-to-right on the tray.

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Here is the setup. I'm going to start by analyzing the whole structure as a rigid body. Before that, I want to label all my joints: A, B, C, D, and E. These lower points are not joints but points of contact with the ground.

Next, I want to identify any two-force members, but there are none. Member AB has pin joints at both ends but also a force in the middle, so it is not a two-force member. Member AC has a connection at the top, a connection in the middle, and a force at the base. The same is true for members BC and CD.

1:33

I also want to check if the structure is independently rigid. In this case, it is. If I picked it up, it would behave as one solid piece. Even if the real tray folds, the way it is drawn here-with all pin joints-does not allow motion. Because it is independently rigid, I can draw a free-body diagram of the entire structure.

2:06

Here is a sketch of the whole TV tray. The forces acting are the 100-pound load at the top and the reaction forces at D and E, which I call F_{Dy} and F_{Ey}. We assume no friction. The distances are one foot from D to the load, and another foot from the load to E.

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Writing equilibrium equations:
- Sum of forces in x: $0 = 0$.
- Sum of forces in y: $F_{Dy} + F_{Ey} - 100 = 0$.
- Sum of moments about point D: $100(1\text{ ft}) + F_{Ey}(2\text{ ft}) = 0$.

Solving gives $F_{Dy} = 50$ lb and $F_{Ey} = 50$ lb. The design is symmetric, so the load splits evenly.

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Next, I draw free-body diagrams of each component: the tabletop and the two legs. The tabletop carries the 100-pound load, and each leg carries a 50-pound reaction at its base.

Each joint is modeled as a pin, so each has forces in the x and y directions. At joint A, I label forces F_{Ax} and F_{Ay}. At joint B, forces F_{Bx} and F_{By}. At joint C, forces F_{Cx} and F_{Cy}.

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Using Newton's third law, each pair of forces at a shared joint must be equal and opposite. I draw the corresponding opposite forces on the legs. This completes the free-body diagrams for all three pieces.

There are six unknowns total: F_Ax, F_Ay, F_Bx, F_By, F_Cx, and F_Cy. I need six equations to solve for them.

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I start with leg BCD.
- Sum of forces in x: F_Cx + F_Bx = 0.
- Sum of forces in y: 50 + F_Cy + F_By = 0.

For moments about point C, I use the distances: one foot horizontally from D to C, one foot from C to B, and 1.5 feet vertically between B and C.

Moment equation about C:
-50(1) + F_By(1) - F_Bx(1.5) = 0.

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Next, I analyze leg AC.
- Sum of forces in x: F_Ax - F_Cx = 0.
- Sum of forces in y: F_Ay - F_Cy = 0.

Moment equation about C:
50(1) - F_Ay(1) - F_Ax(1.5) = 0.

13:24

Now I have six equations and six unknowns. Solving them (using a symbolic solver) gives:
- F_Ax = 66.67 lb
- F_Ay = 50 lb
- F_Bx = -66.67 lb
- F_By = -50 lb
- F_Cx = 66.67 lb
- F_Cy = 0 lb

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The negative signs indicate the actual directions are opposite the assumed directions. For example, F_By and F_Ay are downward, not upward. F_Bx is to the left, not the right.

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Finally, I redraw the solution with corrected directions:

- At C: F_{Cx} = 66.67 lb (horizontal), F_{Cy} = 0.
- At B: 50 lb downward, 66.67 lb to the left.
- At A: 50 lb downward, 66.67 lb to the right.

This completes the worked example.