17.5: The Accelerated Successive Substitution Method (ASSM)

When the system is close to the critical point and fugacities are strongly composition-dependent, a slowing-down of the convergence rate of the SSM (Successive Substitution Method) is to be expected. In an attempt to avoid slow convergence problems, some methods have been proposed. Among the most popular are the Minimum Variable Newton Raphson (MVNR) Method and the Accelerated and Stabilized Successive Substitution Method (ASSM).

The ASSM is basically an accelerated version of the SSM procedure, and thus follows a similar theory. Such procedure is implemented to accelerate the calculation of $K_i$-values, especially in the region close to critical point where the use of the SSM alone will not be efficient. The ASSM technique was presented by Rinses et al. (1981) and consists of the following steps:

1. Use the SSM technique to initiate the updating of the $K_i$-values the first time.
2. Check all following criteria at every step during iterations using the SSM:
   
   Contact your instructor if you are unable to see or interpret this graphic.
   (17.13a)
   \[ |ag_{\text{new}} - ag_{\text{old}}| < 0.1 \] (17.13b)

   Contact your instructor if you are unable to see or interpret this graphic.
   (17.13c)
   \[ 0 < ag_{\text{new}} < 1 \] (17.13d)

   These criteria show that you have sufficient proximity to the conditions to ensure the efficiency of the method. $Rr_i$ is the ratio of liquid fugacity to gas fugacity of the i-th component and ‘$ag$’ is molar gas fraction of the two-phase system.
3. If the system satisfies ALL above criteria, the iteration technique is then switched from the SSM to the ASSM. Otherwise, SSM is used for the update of the $K_i$-values. The following expressions are used to update $K_i$-values in ASSM:

$$K_i^{\text{new}} = K_i^{\text{old}} R r_i \lambda_i$$

where $\lambda_i = \left[ \frac{(R r_i^{\text{old}} -1)}{(R r_i^{\text{old}} - R r_i^{\text{new}})} \right]$

In some cases, using a constant acceleration value of $\lambda_i = 2$ is good enough.

4. Once all the criteria in step (2) are satisfied, skip step (2) for the subsequent iterations and use the ASSM technique to update $K_i$-values until convergence is attained, unless it does not give acceptable new estimates (as stated next).

5. When ASSM is used, it must always be tested to show that it leads to an improved solution (i.e., that it brings fugacity ratios closer to unity). If not, it must be rejected and switched back to SSM.

Even though we are outlining Risnes et al.'s version of the accelerated Successive Substitution Method, there are several other published algorithms whose main purpose have also been to accelerate the successive substitution method. Fussel and Yanosik (1978), Michelsen (1982), and Mehra et al. (1983) are examples of such attempts. Risnes et al. version is the easiest and most straightforward to implement, but it is subjected to limitations.

Contributors

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