1.9: Influence Lines for Statically Determinate Structures

Chapter 9

Influence Lines for Statically Determinate Structures

9.1 Introduction

Structures such as bridges and overhead cranes must be designed to resist moving loads as well as their own weight. Since structures are designed for the critical loads that may occur in them, influence lines are used to obtain the position on a structure where a moving load will cause the largest stress. Influence lines can be defined as a graph whose ordinates show the variation of the magnitude of a certain response function of a structure as a unit load traverses across the structure. Response functions of a structure may include axial forces in members, support reactions, bending moments, shear forces, and deflection at specific points in the structure.

It is very important to emphasize the need for students to fully grasp the afore-stated definition, since most of the confusion and difficulty encountered when drawing influence lines stems from a lack of understanding of the difference between this topic and the bending moment and shearing force topics detailed in chapter four. A shearing force or bending moment diagram shows the magnitude of the shearing force or bending moments at different points of the structure due to the static or stationary loads that are acting on the structure, while the influence lines for certain functions of a structure at a specified point of the structure show the magnitude of that function at the specified point when a unit moving load traverses across the structure. The influence lines of determinate structures can be obtained by the static equilibrium method or by the kinematic or Muller-Breslau method. Influence lines by the static equilibrium method are referred to as quantitative influence lines, as they require some calculations, while those by kinematic method are known as the qualitative influence lines, as the method enables the analyzer to obtain the correct shape of the influence lines without any quantitative efforts. In the subsequent sections, students will consider how to construct the influence lines for beams and trusses using these two methods.
9.2 Influence Lines for Statically Determinate Beams by Static Equilibrium Method

To grasp the basic concept of influence lines, consider the simple beam shown in Figure 9.1a. Statics help to determine the magnitude of the reactions at supports A and B, and the shearing force and bending moment at a section n, as a unit load of arbitrary unit, moves from right to left.

![Simple Beam Diagram](https://eng.libretexts.org/Bookshelves/Civil_Engineering/Book%3A_Structural_Analysis_(Udoeyo)/01%3A_Chapters/1.09%3A...)

Fig. 9.1a. Simple beam.

9.2.1 Beam Reactions

Taking the moment about B as the unit load moves a distance x from the right-hand end suggests the following:

\[
\begin{align*}
+ \sum M_B &= 0 \\
- R_A L + P x &= 0 \\
R_A &= \frac{P}{L} 
\end{align*}
\]  

(9.1)

Setting \( P = 1 \) suggests the following:

\[
R_A = \frac{x}{L}
\]  

(9.2)

Equation 9.2 is the expression for the computation of the influence line for the left-end reaction of a simply supported beam. The influence line for \( R_A \) can be represented graphically by putting some values of x into the equation. Since the equation is linear, two points should be enough.

When \( x = 0, R_A = 0 \)

When \( x = L, R_A = 1 \)

The graphical representation of the influence line for \( R_A \) is shown in Figure 9.1b, and the ordinate of the diagram corresponding to any value of x gives the magnitude of \( R_A \) at that point.
Similarly, the expression for the influence line for the reaction $R_B$ is found by taking the moment about $A$.

$$\sum M_A = 0$$

$$R_B L - P(L - x) = 0$$

$$R_B = \frac{P(xL)}{L}$$

(9.3)

Setting $P = 1$ into equation 9.3 suggests the following:

$$R_B = \frac{(L-x)}{L}$$

(9.4)

Equation 9.4 is the expression for the computation of the influence line for the right-end reaction of a simply supported beam. Substituting some values for $x$ into the equation helps to construct the influence line diagram for $R_B$.

When $x = 0$, $R_B = 1$

When $x = L$, $R_B = 0$

The graphical representation of the influence line for $R_B$ is shown in Figure 9.1c.

9.2.2 Shearing Force at Section $n$

When the unit load is on the right side of the section, the shear force at the section can be computed considering the transverse forces on the left side of the section, as follows:

Shearing force, $V = R_A = \frac{x}{L}$

When $x = 0$, $V = 0$

When $x = b$, $V = \frac{b}{L}$

When the unit load is on the left side of the section, it is easier to compute the shear force in the section by considering the forces on the right side of section, as follows:
\[
V = -R_B = \frac{-(L-x)}{L}
\]
When \( x = b \), \( V = \frac{-(L-b)}{L} = -\frac{a}{L} \)
When \( x = L \), \( V = 0 \)

The graphical representation of the influence line for the shearing force at a section \( n \) of the simple beam is shown in Figure 9.1d.

9.2.3 Bending Moment at a Section \( n \)

When the unit load is on the right side of the section, the bending moment at the section can be computed as follows:

\[
M = R_A (L-x) = \frac{x}{L} (L-x)
\]
When \( x = 0 \), \( M = 0 \)
When \( x = b \), \( M = \frac{ab}{L} \)

When the unit load is on the left side of section, the bending moment at the section can be computed as follows:

\[
M = R_B x = \frac{(L-x)}{L} x
\]
When \( x = 0 \), \( M = 0 \)
When \( x = b \), \( M = \frac{ab}{L} \)

The graphical representation of the influence line for the bending moment at a section \( n \) of the simple beam is shown in Figure 9.1e.
9.3 Construction of Influence Lines

In practice, influence lines are mostly constructed, and the values of the functions are determined by geometry. The procedure for the construction of influence lines for simple beams, compound beams, and trusses will be outlined below and followed by a solved example to clarify the problem. For each case, one example will be solved immediately after the outline.

9.3.1 Simple Beams Supported at Their Ends

The procedures for the construction of the influence lines (I.L.) for some functions of a beam supported at both ends are as follows:

9.3.1.1 Influence Line for Left End Support Reaction, $R_A$ (Fig. 9.2)

(a) At the position of the left end support (point $A$), along the $y$-axis, plot a value +1 (point $A'$).

(b) Draw a line joining point $A'$ and the zero ordinate at point $B$. Point $B$ is at the position of support $B$.

(c) The triangle $AA'B$ is the influence line for the left-end support reaction. The idea here is that when the unit load moves across the beam, its maximum effect on the left-end reaction will be when it is directly lying on the left end support. As the load moves away from the left end support, its influence on the left end reaction will continue to diminish until it gets to the least value of zero, when it is lying directly on the right end support.

Fig. 9.2. Influence line for $R_A$.

9.3.1.2 Influence Line for Right End Support Reaction $R_B$ (Fig. 9.3)

(a) At the right end support (point $B$), plot an ordinate of value +1 (point $B'$).
(b) Draw a line joining point $B'$ and point $A$.

(c) The triangle $AB'B$ is the influence line for the right end support reaction. The explanation for the influence line for the right end support reaction is similar to that given for the left end support reaction. The maximum effect of the unit load occurs when it is lying directly on the right support. As the load moves away from the right end support, its influence on the support reaction decreases until it is zero, when the load is directly lying on the left support.

![Influence line for $R_B$.](image)

**Fig. 9.3. Influence line for $R_B$.**

### 9.3.1.3 Influence Line for Shearing Force at Section $n$

(a) At the left end support (point $A$), plot an ordinate equal +1 (point $A'$), as shown in Figure 9.4b.

(b) Draw a line joining point $A'$ and the zero ordinate at point $B$.

(c) At the right end support (point $B$), plot an ordinate equal $-1$ (point $B'$).

(d) Draw a line joining $B'$ and the zero ordinate at point $A$.

(e) Drop a vertical line from the section under consideration to cut lines $A'B$ and $AB'$ at points $N'$ and $N''$, respectively.

(f) The diagram $ABNN''$ is the influence line of the shear force at the section $n$.

(g) Use a similar triangle to determine the ordinates $n-N'$ and $n-N''$, as follows:
\[
\frac{1}{L} = \frac{n-N'}{b} \rightarrow N' = \frac{b}{L}
\]

\[
\frac{-1}{L} = \frac{n-N''}{a} \rightarrow N'' = -\frac{a}{L}
\]

Fig. 9.4. Influence line for shear (b) and moment (c) at section \( m \).

9.3.1.4 Influence Line for Bending Moment at Section \( n \)

(a) At the left end support (point \( A \)), plot an ordinate of a value equal to the distance from the left end support to the section \( n \). For example, the distance \( a \) in Figure 9.4c (denoted as point \( Y \) in Figure 9.4c).

(b) Draw a line joining point \( Y \) and the zero ordinate at point \( B \) at the right end support.

(c) Draw a vertical line passing through section \( n \) and intersecting the line \( AZ \) at point \( Q \).

(d) Draw a straight line \( AQ \) connecting \( A \) and \( Q \).

(e) The triangle \( AQB \) is the influence line for the moment at section \( n \). Alternatively, ignore steps (b), and (c) and (d) and go to step (f).

(f) At the right end support (point \( B \)), plot an ordinate equal +b. For example, the distance from the right end support to the section \( n \) (denoted as point \( Z \)).

(g) Draw a line joining \( Z \) and the zero ordinate at \( A \) (position of the left end support).
(h) At the left end support (point A), plot an ordinate equal +a. For example, the distance from the left end support to the section n (denoted point Y).

(i) Draw a line joining Y and the zero ordinate at B (position of the right end support).

(j) Lines AZ and BY intersect at Q.

(k) The triangle AQY is the influence line for the moment at section n. If accurately drawn, with the right sense of proportionality, the intersection Q should lie directly on a vertical line passing through the section n.

(l) The value of the ordinate nQ can be obtained using a similar triangle, as follows:

\[
\frac{a}{L} = \frac{nQ}{b} \rightarrow nQ = \frac{ab}{L}
\]

or

\[
\frac{b}{L} = \frac{nQ}{a} \rightarrow nQ = \frac{ab}{L}
\]

Example 9.1

For the double overhanging beam shown in Figure 9.5a, construct the influence lines for the support reactions at B and C and the shearing force and the bending moment at section n.

fig9-5.jpg

Fig. 9.5. Double overhanging beam.
Solution

I.L. for $B_y$.

Step 1. At the position of support $B$ (point $B$), plot an ordinate $+1$.

Step 2. Draw a straight line connecting the plotted point ($+1$) to the zero ordinate at the position of support $C$.

Step 3. Continue the straight line in step 2 until the end of the overhangs at both ends of the beam. The influence line for $B_y$ is shown in Figure 9.5b.

Step 4. Determine the ordinates of the influence line at the overhanging ends using a similar triangle, as follows:

Ordinate at $A$:

$$\frac{1}{BC} = \frac{x}{AC}; \Rightarrow x = \frac{AC}{BC} = \frac{8}{4} = 2 \text{ m}$$

Ordinate at $D$:

$$\frac{1}{BC} = \frac{x}{CD}; \Rightarrow x = \frac{CD}{BC} = \frac{4}{4} = 1 \text{ m}$$

I.L. for $C_y$.

Step 1. At the position of support $C$ (point $C$), plot an ordinate $+1$.

Step 2. Draw a straight line connecting the plotted point ($+1$) to the zero ordinate at the position of support $B$.

Step 3. Continue the straight line in step 2 until the end of the overhangs at both ends of the beam. The influence line for $B_y$ is shown in Figure 9.5c.
Step 4. Determine the ordinates of the influence line at the overhanging ends using a similar triangle, as follows:

Ordinate at $D$:

$$\frac{1}{BC} = \frac{x}{BD}; \Rightarrow x = \frac{BD}{BC} = \frac{8}{4} = 2 \text{ m}$$

Ordinate at $A$:

$$\frac{x}{AB} = \frac{1}{BC}; \Rightarrow x = \frac{AB}{BC} = \frac{4}{4} = 1 \text{ m}$$

I.L. for shear $V_n$.

Step 1. At the position of support $B$ (point B), plot an ordinate $+1$.

Step 2. Draw a straight line connecting the plotted point $(+1)$ to the zero ordinate at the position of support $C$. Continue the straight line at $C$ until the end of the overhang at end $D$.

Step 3. At the position of support $C$ (point C), plot an ordinate $-1$.

Step 4. Draw a straight line connecting the plotted point $(-1)$ to the zero ordinate at the position of support $B$. Continue the straight line at $B$ until the end of the overhang at end $A$.

Step 5. Draw a vertical passing through the section whose shear is required to intersect the lines in step 2 and step 3.

Step 6. Connect the intersections to obtain the influence line, as shown in Figure 9.5d.

Step 7. Determine the ordinates of the influence lines at other points by using similar triangles, as previously demonstrated.

I.L. for Moment $M_n$.

Step 1. At point $B$, plot the ordinate equal $+2 \text{ m}$.

Step 2. Draw a straight line connecting the plotted ordinate in step 1 to the zero ordinate in support C.

Step 3. At point $C$, plot the ordinate equal $+2 \text{ m}$.

Step 4. Draw a straight line connecting the plotted ordinate in step 3 to the zero ordinate at support $B$.

Step 5. Continue the straight lines from the intersection of the lines drawn in steps 2 and 4 through the supports to the overhanging ends, as shown in Figure 9.5e.

Step 6. Determine the values of the influence lines at other points using similar triangles, as previously demonstrated.

Example 9.2

For the beam with one end overhanging support $B$, as shown in Figure 9.6, construct the influence lines for the bending
moment at support $B$, the shear force at support $B$, the support reactions at $B$ and $C$, and the shearing force and the bending moment at a section "$k$.”

Fig. 9.6. Beam with one overhanging support.

**Solution**

The influence lines in example 9.2 for the desired functions were constructed based on the procedure described in the previous section and example.

9.3.2 Compound Beams

To correctly draw the influence line for any function in a compound beam, a good understanding of the interaction of the members of the beam is necessary, as was discussed in chapter 3, section 3.3. The student should recall from the previous section that a compound beam is made up of the primary structure and the complimentary structure. The two facts stated below must always be remembered, since the extent of the spread of the influence line of compound beams depends on them. Remembering these facts will also serve as a temporary check to ascertain the correctness of the drawn influence line.

The moving unit load will have an effect on the functions of the primary structure when it is located at any point, not only
on the primary structure but also on the complimentary structure, since the latter constitutes a loading on the former.

The moving unit load will have effect only on the functions of the complimentary structure when it is located within the complimentary structure; it will not have an effect on any function of the complimentary structure when it is at any point on the primary structure.

The afore-stated facts will be demonstrated in the following examples.

Example 9.3

For the compound beam shown in Figure 9.7, construct the influence lines and indicate the critical ordinates for the support reactions at $A$, $B$, and $D$, the bending moment at $B$, and the shear at hinge $C$.

Fig. 9.7. Compound beam.

Solution

Prior to the construction of the influence lines for desired functions, it is necessary to first observe the extent of the influence lines through the schematic diagram of member-interaction, as shown in Figure 9.7b.

I.L. for $A_y$. The reaction $A_y$ is a function in the primary structure, so the unit load will have influence on this function when it is located at any point on the beam, as was previously stated in section 9.3.2. With this understanding, construct the influence line of $A_y$, as follows:

Step 1. At point $A$, plot an ordinate +1.
Step 2. Draw a straight line connecting the plotted ordinate in step 1 to the zero ordinate in support $B$ and continue this line until the end of the overhanging end of the primary structure, as shown in the interaction diagram.

Step 3. Draw a straight line connecting the ordinate at the end of the overhang to the zero ordinate at support $D$. The influence line is as shown in Figure 9.7c.

Step 4. Use a similar triangle to compute the ordinates of the influence line

I.L. for $B_y$. The influence line for this reaction will cover the entire length of the beam because it is a support reaction in the primary structure. With this knowledge, construct the influence line for $B_y$, as follows:

Step 1: At point $B$, plot an ordinate +1.

Step 2. Draw a straight line connecting the plotted ordinate in step 1 to the zero ordinate in support $A$. Continue the line in support $B$ until the end of the overhanging end of the primary structure, as shown in the interaction diagram.

Step 3. Draw a straight line connecting the ordinate at the overhanging end to the zero ordinate at support $D$. The influence line for $B_y$ is shown in Figure 9.7d.

Step 4. Use a similar triangle to determine the values of the ordinate of the influence line.

I.L. for $D_y$. The reaction $D_y$ is a function in the complimentary structure and will be influenced when the unit load lies at any point along the complimentary structure. It will not be influenced when the unit load transverses the primary structure, as was stated in section 9.3.2. Thus, the extent of the influence line will be the length of the complimentary structure. Knowing this, draw the influence line for $D_y$.

Step 1. At point $D$, plot the ordinate +1.

Step 2. Draw a straight line connecting the plotted ordinate in step 1 to the zero ordinate at hinge $C$. The influence line for $D_y$ is as shown in Figure 9.7e.

The influence lines for the moment at $B$ and the shear $C$ are shown in Figure 9.7f and Figure 9.7g, respectively.

Example 9.4

For the compound beam shown in Figure 9.8a, construct the influence lines and indicate the critical ordinates for the support reactions at $F$ and $G$, the shear force and bending moment at $D$, and the moment at $F$. 

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9.3.3 Influence Lines for Girders Supporting Floor Systems

Thus far, the examples and text have only considered cases where the moving unit load is applied directly to the structure. But, in practice, this may not always be the case. For instance, sometimes loads from building floors or bridge decks are transmitted through secondary beams, such as stringers and cross beams to girders supporting the building or bridge floor system, as shown in Figure 9.9. Columns, piers, or abutments in turn support the girders.

As shown in Figure 9.9, the vehicular load from the bridge deck is transferred to the girder at points 1, 2, 3, 4, and 5, referred to as panel points, where the floor beams are in contact with the girder. The segment between two successive contact points is known as a panel. For an illustration of the construction of influence lines in a case of indirect application of loads, the floor beams and the girder of Figure 9.9 are separated from the entire system, as shown in Figure 9.10. Assume the length of each panel equals 4m. Construct the influence lines for the moment at point 4 and for the moment and shear at a section \( r \) at the midpoint of 3 and 4 (a point lying within panel 3-4). The influence line for the moment at point 4 is shown in Figure 9.10b; notice that the construction of
the influence line for moment at this point is exactly like the cases considered in previous sections, where the moving load is applied directly to the beam. When the unit load moves to the right of 4 and to the left of 3, the influence line for the moment for any section within panel 3-4 will be constant, as shown in Figure 9.10c. The construction of the influence line for the shear of any section within the panel 3-4 is obtained in the same manner as when the unit load is directly applied to the girder, with the exception that a diagonal line is drawn to connect the points where a vertical line drawn from the points intersect with the construction line.

![Influence line for moment and shear](https://eng.libretexts.org/Bookshelves/Civil_Engineering/Book%3A_Structural_Analysis_(Udoeyo)/01%3A_Chapters/1.09%3A...

Fig. 9.10. Influence lines in a case of indirect application of loads.

**Example 9.5**

Draw the influence lines for the moment at C and the shear in panel BC of the floor girder shown in Figure 9.11.

![Floor girder](https://eng.libretexts.org/Bookshelves/Civil_Engineering/Book%3A_Structural_Analysis_(Udoeyo)/01%3A_Chapters/1.09%3A...)

Fig. 9.11a. Floor girder.

**Solution**

Influence line for $M_C$. To obtain the values of the influence line of $M_C$, successively locate a load of 1 kN at panel points A, B, C, D, and E. To determine the moment, use the equation of statics. The values of $M_C$ at the respective panel...
points are presented in Table 9.1. When the unit load is located at B, as shown in Figure 9.11b, the value of $M_C$ is determined as follows:

![Fig. 9.11b. Unit load at B.](image)

First, determine the support reactions in the beam using the equation of static equilibrium.

\[
\begin{align*}
\sum F_x &= 0 & \rightarrow & -A_y (16) + 1(12) = 0 & A_y = 0.75 \text{ kN} \\
\sum F_y &= 0 & \rightarrow & 0.75 + E_y - 1 = 0 & E_y = 0.25 \text{ kN}
\end{align*}
\]

Then, using the computed reaction, determine $M_C$, as follows:

$M_C = 0.25(12) = 3 \text{ kN} \cdot \text{m}$

Table 9.1. The values of $M_C$ at the respective panel points.

<table>
<thead>
<tr>
<th>$x$(m)</th>
<th>Reactions(kN)</th>
<th>$M_C$(kN. m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0.75</td>
<td>0.25(8) = 2</td>
</tr>
<tr>
<td>8</td>
<td>0.5</td>
<td>0.5(8) = 4</td>
</tr>
<tr>
<td>12</td>
<td>0.25</td>
<td>0.25(8) =2</td>
</tr>
<tr>
<td>16</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

![Graph](image)
Fig. 9.11c. Influence line for $M_C$.

Influence line for $V_{BC}$. To obtain the values of the influence line of $MV_{BC}$, a load of 1 kN is successively located at panel points $A$, $B$, $C$, $D$, and $E$. To determine the shear force, use the equation of statics. The values of $V_C$ at the respective panel points are presented in Table 9.2.

Table 9.2. The values of $V_C$ at the respective panel points.

<table>
<thead>
<tr>
<th>$x$ (m)</th>
<th>$A_y$ (kN)</th>
<th>$E_y$ (kN)</th>
<th>$V_{BC}$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0.75</td>
<td>0.25</td>
<td>$-0.25$</td>
</tr>
<tr>
<td>8</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>12</td>
<td>0.25</td>
<td>0.75</td>
<td>0.25</td>
</tr>
<tr>
<td>16</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 9.2. The values of $V_C$ at the respective panel points.

Fig. 9.11d. Influence line for $V_{BC}$.

9.3.4 Influence Lines for Trusses

The procedure for the construction of influence lines for truss members is similar to that of a girder supporting a floor system considered in section 9.3.3. Loads can be transmitted to truss members via the top or bottom panel nodes. In Figure 9.12 the load is transmitted to members through the top panel nodes. As the live loads move across the truss, they are transferred to the top panel nodes by cross beams and stringers. The influence lines for axial forces in truss members can be constructed by connecting the influence line ordinates at the panel nodes with straight lines.
To illustrate the procedure for the construction of influence lines for trusses, consider the following examples.

Example 9.6

Draw the influence lines for the reactions $A_y$, $F_y$, and for axial forces in members $CD$, $HG$, and $CG$ as a unit load moves across the top of the truss, as shown in Figure 9.13.

Solution

The drawing of the influence lines for trusses is similar to that of a beam. The first step towards drawing the influence lines for the axial forces in the stated members is to pass an imaginary section through the members, as shown in Figure 9.13b, and to apply equilibrium to the part on either side of the section. The step-by-step procedure for drawing the influence line for each of the members is stated below.

Influence line for the axial force in member $CD$. When the unit load is situated at any point to the right of $D$, considering the equilibrium of the left segment $AH$ (Fig. 9.13c), it suggests the following:
The obtained expression of $F_{CD}$ in terms of $A_y$ is indicative of the fact that the influence line for $F_{CD}$ in the portion $DE$ can be determined by multiplying the corresponding portion of the influence line for the reaction $A_y$ by $-2$. The influence line for $A_y$ is shown in Figure 9.13e.

When the unit load is situated at any point to the left of $C$, considering the equilibrium of the right segment $GF$ (Fig. 9.13d), it suggests the following:

$$+\sum M_c = 0 \quad -A_y(6) - F_{CD}(3) = 0 \quad F_{CD} = -2A_y$$

The obtained expression of $F_{CD}$ in terms of $F_y$ is indicative of the fact that the influence line for $F_{CD}$ in the portion $AH$ can be determined by multiplying the corresponding portion of the influence line for the reaction $F_y$ by $-1$. The influence line for $F_y$ is shown in Figure 9.13f.

The influence line of the axial force in member $CD$ constructed from the influence lines for the reactions $A_y$ and $F_y$ is shown in Figure 9.13g.

Influence line for member $HG$. When the unit load is situated at any point to the right of $D$, considering the equilibrium of the left segment $AH$ (Fig. 9.13c), it suggests the following:

$$+\sum M_c = 0 \quad -A_y(3) + F_{HG}(3) = 0 \quad F_{HG} = A_y$$

The obtained expression of $F_{HG}$ in terms of $A_y$ implies that the influence line for $F_{HG}$ in the portion $DE$ is identical to that of $A_y$ within the corresponding segment.

When the unit load is situated at any point to the left of $C$, considering the equilibrium of the right segment $GF$ (Fig. 9.13d), it suggests the following:

$$+\sum M_c = 0 \quad F_y(6) - F_{HG}(3) = 0 \quad F_{HG} = 2F_y$$

The obtained expression of $F_{HG}$ in terms of $F_y$ is indicative of the fact that the influence line for $F_{HG}$ in the portion $AH$ can be determined by multiplying the corresponding portion of the influence line for the reaction $F_y$ by $2$.

The influence line of the axial force in member $HG$ constructed from the influence line for the reactions $A_y$ and $F_y$ is also shown in Figure 9.13h.

Influence line for the axial force in member $CG$. When the unit load is situated at any point to the right of $D$, considering the equilibrium of the left segment $AH$ (Fig. 9.13c), it suggests the following:

$$+\sum F_y = 0 \quad A_y - F_{CG} \cos 45^\circ = 0 \quad F_{CG} = \frac{A_y}{\cos 45^\circ} = 1.41A_y$$

The obtained expression of $F_{CG}$, with reference to $A_y$, implies that the influence line for $F_{CG}$ in the portion $DE$ can be determined by multiplying the corresponding portion of the influence line for the reaction $A_y$ by $1.41$. 

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When the unit load is situated at any point to the left of C, considering the equilibrium of the right segment GF (Fig. 9.13d), it suggests the following:

$$F_y + F_{CG} \cos 45^\circ = 0$$

$$F_{CG} = -\frac{F_y}{\cos 45} = -1.41F_y$$

The obtained expression of $F_{CG}$ in terms of $F_y$ is indicative of the fact that the influence line for $F_{CG}$ in the portion AH can be determined by multiplying the corresponding portion of the influence line for the reaction $F_y$ by – 1.41.

The influence line of the axial force in member CG constructed from the influence line for the reactions $A_y$ and $F_y$ is shown in Figure 9.13i.

Example 9.7

Draw the influence lines for the force in member CH as a unit load moves across the top of the truss, as shown in Figure 9.14a.

**Solution**

To obtain the expression for the influence line for the axial force in member CH, first pass an imaginary section that cuts through this member, as shown in Figure 9.14a.

When the unit load is situated at any point to the right of G, considering the equilibrium of the left segment AH (Fig. 9.14c), it suggests the following:

$$\sum F_y = 0 \quad A_y + F_{CH} = 0 \quad F_{CH} = -A_y$$
The obtained expression of \( F_{CH} \) in terms of \( A_y \) indicates that the influence line for \( F_{CH} \) in the portion \( AH \) can be determined by multiplying the corresponding portion of the influence line for the reaction \( A_y \) by \(-1\).

When the unit load is located at any point to the left of \( H \), considering the equilibrium of the right segment \( GF \) (Fig. 9.14d), it suggests the following:

\[ \sum F_y = 0 \quad F_y - F_{CH} = 0 \quad F_{CH} = F_y \]

The obtained expression of \( F_{CH} \) in terms of \( F_y \) implies that the influence line for \( F_{CH} \) in the portion \( GF \) is identical to that of \( F_y \) within the corresponding segment.

The influence line of \( CG \) is shown in Figure 9.14g.

### 9.4 Uses of Influence Lines

#### 9.4.1 Uses of Influence Lines to Determine Response Functions of Structures Subjected to Concentrated Loads

The magnitude of a response function of a structure due to concentrated loads can be determined as the summation of the product of the respective loads and the corresponding ordinates of the influence line for that response function. Example 9.5 and example 9.6 illustrate such cases.

Example 9.8

A simple beam is subjected to three concentrated loads, as shown in Figure 9.15a. Determine the magnitudes of the reactions and the shear force and bending moment at the midpoint of the beam using influence lines.
Fig. 9.15. Simple beam.

Solution

First, draw the influence line for the support reactions and for the shearing force and the bending moment at the midpoint of the beam (see Fig. 9.15b, Fig. 9.15c, Fig. 9.15d, and Fig. 9.15e). Once the influence lines for the functions are drawn, compute the magnitude of the response functions, as follows:

Magnitude of the support reactions using the influence line diagrams in Figure 9.15b and Figure 9.15c.

\[
A_y = (12)\left(\frac{2}{6}\right) + (14)\left(\frac{2}{3}\right) + (16)\left(\frac{1}{3}\right) = 24.67 \text{ kN}
\]

\[
E_y = (12)\left(\frac{1}{6}\right) + (14)\left(\frac{1}{3}\right) + (16)\left(\frac{2}{3}\right) = 17.33 \text{ kN}
\]

Magnitude of the shear force at section \(n\) using the influence line diagram in Figure 9.15d.

\[
V_n = (12)\left(-\frac{1}{6}\right) + (14)\left(-\frac{1}{3}\right) + (16)\left(\frac{1}{3}\right) = -1.33 \text{ kN}
\]

Magnitude of the bending moment at section \(n\) using the influence line diagram of Figure 9.15e.

\[
M_n = (12)\left(\frac{2}{2}\right) + (14)(3) + (16)(3) = 108 \text{ kN.m}
\]

Example 9.9

A compound beam is subjected to three concentrated loads, as shown in Figure 9.16a. Using influence lines, determine the magnitudes of the shear and the moment at \(A\) and the support reaction at \(D\).
Solution

First, draw the influence line for the shear force $V_A$, bending moment $M_A$, and reaction $C_y$. The influence lines for these functions are shown in Figure 9.16b, Figure 9.16c, and Figure 9.16d. Then, compute the magnitude of these response functions, as follows:

The magnitude of the shear at section $n$ using the influence line diagram in Figure 9.16b.

$$V_A = (8)(1) + (12)\left(\frac{1}{2}\right) + -(12)(1) = 26 \text{ kips}$$

The magnitude of the bending moment at section $n$ using the influence line diagram in Figure 9.16c.

$$M_A = (8)(-6) + (12)(-3) + (12)(6) = -12 \text{ kip-ft}$$

Magnitude of the support reaction $C_y$ using the influence line diagram in Figure 9.16d.

$$C_y = (12)\left(\frac{1}{2}\right) + (12)(2) = 30 \text{ kips}$$
The magnitude of a response function of a structure subjected to distributed loads can be determined as the product of the intensity of the distributed load and the area of the influence line. Consider a beam subjected to a uniform load \( \omega_x \), as shown in Figure 9.17a. First, convert the uniform load to an equivalent concentrated load. The equivalent elementary concentrated load for a distributed load acting on a differential length \( dx \) is as follows:

\[
dP = \omega_x dx
\]

(9.5)

The magnitude of the response function \( (rf) \) due to the elementary concentrated load acting on the structure can be expressed as follows:

\[
rf = (\omega_x dx)(y)
\]

(9.6)

where

\[y = \text{the ordinate of the influence line at the point of application of the load } dP.\]

![Fig. 9.17. Beam subjected to uniform load.](https://eng.libretexts.org/Bookshelves/Civil_Engineering/Book%3A_Structural_Analysis_(Udoeyo)/01%3A_Chapters/1.09%3A_...)

The total response function (\( RF \)) due to the distributed load acting at the segment \( BC \) of the beam is obtained by integration, as follows:

\[
RF = \int_b^C (\omega_x)(y)dx = \omega_x \int_b^C (y)dx
\]

(9.7)

The integral
\[ \int_{B}^{C} (y) \, dx \]

is the area under the portion of the influence line corresponding to the loaded segment of the beam (see the shaded area in Fig. 9.17b).

Example 9.10

Using influence lines, determine the shear force and the bending moment at the midpoint of the loaded simple beam, as shown in Figure 9.18a.

Fig. 9.18. Loaded simple beam.

Solution

First, draw the influence line for the shear force and bending moment at the midspan of the beam. The influence lines for these functions are shown in Figure 9.18b and Figure 9.18c. Then, compute the magnitude of these response functions, as follows:

From the influence line diagram, as shown in Figure 9.18b, the magnitude of the shear at B is as follows:

\[ V_B = (2)\left(-\frac{1}{2} \times 6 \times \frac{1}{2}\right) + (1.5)\left(\frac{1}{2} \times 6 \times \frac{1}{2}\right) = -0.75 \text{ kip} \]

The magnitude of the bending moment at point B, using influence line diagram in Figure 9.18c, is as follows:

\[ M_B = (2)\left(\frac{1}{2} \times 6 \times 3\right) + (1.5)\left(\frac{1}{2} \times 6 \times 3\right) = 31.5 \text{ kip-ft} \]

Example 9.11
A compound beam is subjected to a combined loading, as shown in Figure 9.19a. Using influence lines, determine the magnitudes of the reactions at supports A, B, and C.

![Compound Beam](image)

**Solution**

The magnitude of the support reaction $A_y$, using the influence line diagram in Figure 9.19b.

$$A_y = (10)(\frac{1}{2})(2)(1) + (10)(\frac{1}{2})(1)(-\frac{3}{4}) + (20)(-\frac{1}{4}) = 2.5 \text{ kN}$$

The magnitude of the support reaction $B_y$, using the influence line diagram in Figure 9.19c.

$$B_y = (10)(\frac{1}{2})(3)(\frac{3}{2}) + (20)(\frac{3}{4}) = 37.5 \text{ kN}$$

The magnitude of the support reaction $D_y$, using the influence line diagram of Figure 9.19d.

$$D_y = (20)(\frac{1}{2}) = 10 \text{ kN}$$

### 9.4.3 Use of Influence Lines to Determine the Maximum Effect at a Point Due to Moving Concentrated Loads

In the analysis and design of structures, such as bridges and cranes subjected to moving loads, it is often desirable to find the position of the moving load(s) that will produce a maximum influence at a point. For some structures, this can be determined by mere inspection, while for most others it may require a trial-and-error process using influence lines. Examples 9.12 and 9.13 illustrate the trial-and-error process involved when using influence lines to compute the magnitude of certain functions of a beam subjected to a series of concentrated moving loads.
Example 9.12

Using influence lines, determine the shear force and bending moment at the midpoint \( k \) of a beam shown in Figure 9.20a. The beam is subjected to a series of moving concentrated loads, which are shown in Figure 9.20b.

![Fig. 9.20. Beam.](image)

Solution

Maximum shear \( V_k \) from Figure 9.20c.

- **Case 1:** \( V_k = (6) \left( \frac{1}{3} \right) + (12) \left( \frac{2}{3} \right) + (12) \left( \frac{2}{5} \right) = 7.2 \, k \)

- **Case 2:** \( V_k = (6) \left( \frac{1}{3} \right) + (12) \left( \frac{2}{3} \right) + (12) \left( \frac{2}{5} \right) = 7.8 \, k \)

- **Case 3:** \( V_k = (6) \left( \frac{1}{3} \right) + (12) \left( \frac{2}{3} \right) + (12) \left( \frac{2}{5} \right) = -9.3 \, k \)

Maximum positive shear = 7.8 k

Maximum negative shear = 9.3 k

Maximum moment \( M_k \) from Figure 9.20d.

- **Case 1:** \( M_k = (6)(6) + (12)(10) + (12)(5) = 216 \, k\text{ft} \)

- **Case 2:** \( M_k = (6)(1) + (12)(5) + (12)(10) = 186 \, k\text{ft} \)

Maximum moment \( M_k \) = 216 k\text{ft}

Example 9.13

A compound beam shown in Figure 9.21a is subjected to a series of moving concentrated loads, which are shown in

https://eng.libretexts.org/Bookshelves/Civil_Engineering/Book%3A_Structural_Analysis_(Udoeyo)/01%3A_Chapters/1.09%3A...
Figure 9.21b. Using influence lines, determine the magnitudes of the reactions at supports A, B, and C and the bending moment at section n.

Solution

Maximum positive reaction $A_x$ from Figure 9.21c.

$$A_x = (60)(1) + (40)(3/2) = 70 \text{ kN} \quad \text{Ans}$$

Maximum reaction $B_y$ from Figure 9.21d.

Case 1: $B = (40)\left(\frac{3}{2}\right) + (60)\left(\frac{1}{2}\right) + (10)(0) = 120 \text{ kN}$

Case 2: $B = (10)\left(\frac{3}{2}\right) + (60)\left(\frac{1}{2}\right) + (40)\left(\frac{1}{2}\right) = 107.5 \text{ kN}$

Maximum $B_y = 120 \text{ kN}$

Maximum positive reaction $D_y$ from Figure 9.21e.

$$D_y = (60)(1) + (40)(0) = 60 \text{ kN}$$

Maximum positive moment $M_n$ from Figure 9.21f.

$$M_n = (60)(2) + (10)(0) = 120 \text{ kN} \cdot \text{m}$$

9.4.4 Uses of Influence Lines to Determine Absolute Maximum Response Function at Any Point Along the Structure

The preceding sections explain the use of influence lines for the determination of the maximum response function that may occur at specific points of a structure. This section will explain the determination of the absolute maximum value of a response function that may occur at any point along the entire structure due to concentrated loads exerted by moving loads.
The absolute maximum shear force for a cantilever beam will occur at a point next to the fixed end, while that for a simply supported beam will occur close to one of its reactions. The absolute maximum moment for a cantilever beam will also occur close to the fixed end, while that for simply supported beam is not readily known and, thus, will require some analysis. To locate the position where the absolute maximum moment occurs in a simply supported beam, consider a beam subjected to three moving concentrated loads $P_1$, $P_2$, and $P_3$, as shown in Figure 9.22.

Although it is certain from statics that the absolute maximum moment will occur under one of the concentrated loads, the specific load under which it will occur must be identified, and its location along the beam must be known. The concentrated load under which the absolute maximum moment will occur may be determined by inspection or by trial-and-error process, but the location of this load should be established analytically. Assume that the concentrated load under which the absolute maximum moment will occur is $P_3$, and the distance of $P_3$ from the centerline of the beam is $x$.

To obtain an expression for $x$, first determine the resultant $P_R$ of the concentrated loads, acting at a distance $x'$ from the load $P_3$.

To determine the right reaction of the beam, take the moment about support $A$, as follows:

\[
\Sigma M_A = 0
\]

\[
B_y L = P_R \left[ \frac{L}{2} - (x' - x) \right]
\]

\[
B_y = \frac{P_R}{L} \left[ \frac{L}{2} - (x' - x) \right]
\]  \hspace{1cm} (9.8)

Thus, the bending moment under $M_3$ is as follows:

\[
M_3 = B_y \left( \frac{L}{2} - x \right) = \frac{P_R}{L} \left[ \frac{L}{2} - (x' - x) \right] \left( \frac{L}{2} - x \right)
\]

\[
= P_R \left( \frac{L}{4} + \frac{x'}{2} + \frac{x' - x}{2} \right)
\]  \hspace{1cm} (9.9)
The distance $x$ for which $M_3$ is maximum can be determined by differentiating equation 9.9 with respect to $x$ and equating it to zero, as follows:

\[
\frac{dM_3}{dx} = P_R \left( \frac{x'}{L} - \frac{2x}{L} \right) = 0
\]

\[
\frac{2x}{L} = \frac{x'}{L}
\]

Therefore,

\[
x = \frac{x'}{2}
\]  

Equation 9.10 concludes that the absolute maximum moment in a simply supported beam occurs under one of the concentrated loads when the load under which the moment occurs and the resultant of the system of loads are equidistant from the center of the beam.

Example 9.14

Determine the absolute maximum bending moment in a 16 m-long simply supported girder bridge subjected to a moving truck loading, as shown in Figure 9.23.

![Fig. 9.23. Simply supported girder beam.](https://eng.libretexts.org/Bookshelves/Civil_Engineering/Book%3A_Structural_Analysis_(Udoeyo)/01%3A_Chapters/1.09%3A…)

Solution

Using statics, first determine the value and the position of the resultant of the moving loads.

Resultant load.

\[
P_R = \sum P = 20 + 50 + 90 = 160
\]

Position of the resultant load. To determine the position of the resultant load, take the moment about point $n$, which is directly below the 20 kN load, as follows:
\[ \sum M_n: 160x = (50)(3) + (90)(8) \]
\[ x = 5.44 \text{ m} \]

Fig. 9.24. Resultant and load equidistant from centerline of the beam.

If the absolute maximum moment is assumed to occur under the 50 kN load, the positioning of the resultant and this load equidistant from the centerline of the beam is as shown in Figure 9.24. Before computing the absolute maximum moment, first determine the reaction \( B_y \) using statics.

\[ \sum M_A = 0: -(160)(9.22) + B_y(16) = 0 \]
\[ B_y = 92.2 \text{ kN} \]

The absolute maximum moment under the 50 kN load is as follows:

\[ M_{50} = (92.2)(9.22) - (90)(3.78) = 509.88 \text{ kN. m} \]
If the absolute maximum moment is assumed to occur under the 90 kN load, the positioning of the resultant and this load equidistant from the centerline of the beam will be as shown in Figure 9.25.

Before computing the absolute maximum moment, first determine the reaction $B_y$ using statics.

$$\sum M_A = 0: -160(6.72) + B_y(16) = 0$$

$$B_y = 67.2 \text{ kN}$$

The absolute maximum moment under the 90 kN load is as follows:

$$M_{90} = (67.2)(6.72) = 451.58 \text{ kN.m}$$

From the two possible cases considered in the solution, it is evident that the absolute maximum moment occurs under the 50 kN force.

Chapter Summary

**Influence lines for statically determinate structures:** The effect of a moving load on the magnitude of certain functions of a structure, such as support reactions, deflection, and shear force and moment, at a section of the structure vary with the position of the moving load. Influence lines are used to study the maximum effect of a moving load on these functions for design purposes. The influence lines for determinate structures can be obtained by the static equilibrium method or by the kinematic or Muller-Breslau method. The influence lines by the former method can be determined quantitatively, while those for the latter method can be obtained qualitatively, as have been demonstrated in this chapter. Several example problems are solved showing how to construct the influence lines for beams and trusses using the afore-stated methods.

Practice Problems
9.1 Draw the influence line for the shear force and moment at a section \( n \) at the midspan of the simply supported beam shown in Figure P9.1.

![Simply supported beam](https://eng.libretexts.org/Bookshelves/Civil_Engineering/Book%3A_Structural_Analysis_(Udoeyo)/01%3A_Chapters/1.09%3A…)

Fig. P9.1. Simply supported beam.

9.2 Draw the influence lines for the reaction at \( A \) and \( B \) and the shear and the bending moment at point \( C \) of the beam with overhanging ends, as shown in Figure P9.2.

![Beam with overhang](https://eng.libretexts.org/Bookshelves/Civil_Engineering/Book%3A_Structural_Analysis_(Udoeyo)/01%3A_Chapters/1.09%3A…)

Fig. P9.2. Beam with overhang.

9.3 Draw the influence line for the reactions at the support of the cantilever beam shown in Figure P9.3.

![Cantilever beam](https://eng.libretexts.org/Bookshelves/Civil_Engineering/Book%3A_Structural_Analysis_(Udoeyo)/01%3A_Chapters/1.09%3A…)

Fig. P9.3. Cantilever beam.

9.4 Draw the influence line for the support reactions at \( B \) and \( D \) and shear and bending moments at section \( n \) of the beam shown in Figure 9.4.

![Beam](https://eng.libretexts.org/Bookshelves/Civil_Engineering/Book%3A_Structural_Analysis_(Udoeyo)/01%3A_Chapters/1.09%3A…)

Fig. P9.4. Beam

9.5 Draw the influence lines for support reactions at \( C \) and \( D \) and at point \( B \) of the compound beam shown in Figure P9.5.

![Compound beam](https://eng.libretexts.org/Bookshelves/Civil_Engineering/Book%3A_Structural_Analysis_(Udoeyo)/01%3A_Chapters/1.09%3A…)

Fig. P9.5. Compound beam.
9.6 Draw the influence lines for the shear force and moment at sections ns and k of the compound beam shown in Figure P9.6.

9.7 Determine the absolute maximum bending moment in a 65 ft-long simply supported girder bridge subjected to a moving truck loading, as shown in Figure P9.7.

9.8 Determine the absolute maximum bending moment in a 12 m-long simply supported girder bridge subjected to a moving truck loading, as shown in Figure P9.8.
9.9 Determine the absolute maximum bending moment in a 40 ft-long simply supported girder bridge subjected to a moving truck loading, as shown in Figure P9.9.

![Simply supported girder bridge](image1)

Fig. P9.9. Simply supported girder bridge.

9.10 Determine the absolute maximum bending moment in a 14 m-long simply supported girder bridge subjected to a moving truck loading, as shown in Figure P9.10.

![Simply supported girder bridge](image2)

Fig. P9.10. Simply supported girder bridge.

9.11 Draw the influence lines for the moment at B and the shear force in panel CD of the floor girder shown in Figure P9.11.

![Floor girder](image3)

Fig. P9.11. Floor girder.

9.12 Draw the influence lines for the moment at C and the shear force in panel BC of the floor girder shown in Figure P9.12.
9.13 Draw the influence lines for the moment at $B$ and the shear in panel $CD$ of the floor girder shown in Figure P9.13.

9.14 Draw the influence lines for the moment at $D$ and the shear force in panel $DE$ of the floor girder shown in Figure P9.14.

9.15 Draw the influence lines for the moment at $D$ and the shear force in panel $AB$ of the floor girder shown in Figure P9.15.

9.16 Draw the influence lines for the forces in members $CD$, $CF$, and $GF$ as a unit load moves across the top of the truss, as shown in Figure P9.16.
9.17 Draw the influence lines for the forces in members $DE$, $NE$, and $NM$ as a unit live load is transmitted to the top chords of the truss, as shown in Figure P9.17.

9.18 Draw the influence lines for the forces in members $DE$, $DH$, $IH$, and $HG$ as a unit live load is transmitted to the bottom chords of the truss, as shown in Figure P9.18.

9.19 Draw the influence lines for the forces in members $BC$, $BF$, $FE$, and $ED$ as a unit load moves across the bottom chords of the truss, as shown in Figure P9.19.
Fig. P9.19. Truss.