1.1: Model Variables and Element Types

Modeling of a physical system involves two kinds of variables: flow variables that ‘flow’ through the system components, and across variables that are measured across those components. In electrical circuits, voltage or potential is measured across the circuit nodes, whereas current or electrical charge flows through the circuit branches. In mechanical linkage systems, displacement and velocity are measured across the connecting nodes, whereas force or effort ‘flows’ through the linkages. In the case of thermal and fluid systems, heat and mass serve as the flow variables, while temperature and pressure constitute the across variables.

The relationship between flow and across variables associated with an element in the system defines the type of physical element being modeled. The three basic types are the resistive, inductive, and the capacitive elements. The terminology taken from electrical circuits extends to many other types of physical systems. Let \( q(t) \) denote a flow variable and \( x(t) \) denote an across variable associated with a physical element; then, the element type is defined by their mutual relationships, described as follows:

**The resistive element.** A resistive element is described by the following relation: \( x(t)=k\; q(t), \) \( k \) The flow and across variables for the element vary in proportion to each other. For example, the voltage and current relationship through a resister is described by Ohm’s law: \( V(t)=R\; i(t), \) that states that the voltage across a resistor varies in proportion to the current through the resistor. Similarly, the force–velocity relationship though a mechanical damper is described as: \( v(t)=\frac{1}{b} f(t), \) \( b \) i.e., the velocity increases in proportion to the applied force.

**The capacitive element.** A capacitive element is described by the following relation: \( x(t)=k\int q(t)\; dt+x_{(0)}, \) The across variable varies in proportion to the accumulated amount of the flow variable. Alternatively, the flow variable varies proportionally with the rate of change of the across variable as: \( q(t)=k\frac{dx(t)}{dt}, \) \( k \) i.e., the voltage across the capacitor is proportional to integral of current through it, where the current...
integral represents the accumulation of electrical charge; hence, \(Q=CV\). The inverse relationship is described as: \(i(t)=C\frac{dV}{dt}\). Similarly, the force–velocity relationship that governs an inertial mass element is given as: \(v(t)=\frac{1}{m}\int f(t)\,dt+v_{0}\). The inverse relation is the familiar Newton’s second law of motion: \(f(t)=ma(t)\), where \(a(t)\) denotes the acceleration.

**The inductive element.** An inductive element is described by the following relation: \(x(t)=k\frac{\mathrm{d}q(t)}{\mathrm{d}t}\). Thus, flow variable is obtained by differentiating the across variable. Alternatively, the across variable varies in proportion to the accumulation of the flow variable.

For example, the voltage–current relationship through an inductive coil in an electric circuit is given as: \(V(t)=L\frac{di(t)}{dt}\). The inverse relationship is described as: \(i(t)=\frac{1}{L}\int V(t)\,dt+i_{0}\). Similarly, the force–velocity relationship through a linear spring is given as: \(v(t)=\frac{1}{K}\frac{\mathrm{d}f(t)}{\mathrm{d}t}\). The inverse relation is: \(f(t)=K\int v(t)\,dt+f_{0}\).

While the resistive element dissipates energy, both the capacitive and inductive elements store energy. For example, a capacitor stores electrical energy and a moving mass stores kinetic energy. The energy storage accords memory to the element that accounts for the dynamic system behavior modeled by an ODE.