8.6: Thermoelectric Efficiency

Carnot Efficiency

Many devices convert a temperature difference to another form of energy. For example, thermoelectric devices and pyroelectric devices convert a temperature difference to electricity, and Stirling engines and steam turbines convert a temperature difference to mechanical work. There is a fundamental limit to the efficiency of any device that converts a temperature difference into another form of energy. The Carnot efficiency is the maximum possible efficiency of such an energy conversion process.

Consider a thermoelectric device made from a junction of two materials that converts a temperature difference to electricity using the Seebeck effect. Assume that one end of the device is connected to a heater, and the other end of the device is connected to a heat sink so that it is at a lower temperature. The temperature of the hot side of the device is denoted T_h, and the temperature of the cold side of the device is denoted T_c. Both temperatures are measured in kelvins, K (or another absolute temperature measure such as Rankine). Assume that the only energy conversion process that occurs converts energy from the temperature difference to electricity. Furthermore, assume that energy is continuously supplied from the heater at a constant rate to maintain the hot end of the device at temperature T_h. The heater is supplying heat to the room it is in. However, assume that the room is so large and the amount of heat from the heater is so small that the temperature of the room remains roughly constant. For this reason, we say that the room is a thermodynamic reservoir. Also, assume that we have waited long enough that the temperature of the device has reached a steady state. The temperature is not constant along the length of the device, but it no longer varies with time.

The input to this system is the thermal energy supplied from the heater, E_{in}. The output of this system is the electrical energy extracted out, E_{out}. The device is not used up in the process, so the number of atoms in the

device remains constant. As long as energy is supplied from the heater at a constant rate to maintain the hot side at temperature T_h, we can extract electrical energy out of the system at a constant rate. Heat transfer scientists call this type of process a thermodynamic cycle or a heat engine. A thermodynamic cycle is a sequence of energy conversion processes where the device begins and ends in the same state. In a thermodynamic cycle, energy is supplied in one form and is extracted in another form. The device or mass involved starts and ends in the same state, so the processes can continue indefinitely as long as the input is continually supplied.

How much energy is supplied in to the system from the heater? The amount of energy required to maintain the hot side at temperature T_h is given by

$$E_{in} = k_BT_h. \label{8.6.1}$$

The device is composed of atoms. Each of these atoms has some internal energy. A device at temperature T contains k_BT joules of energy where k_B is the Boltzmann constant. Energy flows from the hot side to the cold side of the device. Above, we assumed that the device was in a room that was so large that the heat from the heater did not raise the temperature of the room. Thus, we must continually supply this energy at a constant rate to keep the hot side of the device at temperature T_h. While the cold side of the device is at a lower temperature T_c, it maintains that temperature regardless of the fact that there is a heater in the room.

How much energy is extracted out of the system as electrical energy? In the Seebeck device, the hot side is held fixed at temperature T_h, and because of the environment it is in, the cold side remains at temperature T_c. Energy is conserved in this system. Thus, the electrical energy extracted from the device is given by

$$E_{out} = k_BT_h - k_BT_c. \label{8.6.2}$$

What is the efficiency of this system? Above we assumed that no other energy conversion processes occur, so this is an idealized case. The resulting efficiency that we calculate represents the best possible efficiency of a thermoelectric device operating with sides at temperatures T_h and T_c. Efficiency is defined as

$$\eta_{ef\;f} = \frac{E_{out}}{E_{in}}. \nonumber$$

Using Equations \ref{8.6.1} and \ref{8.6.2} and some algebra, we can simplify the efficiency expression.

$$\eta_{ef\,f} = \frac{E_{out}}{E_{in}} = \frac{k_BT_h - k_BT_c}{k_BT_h} \nonumber$$

$$\eta_{ef\,f} = \frac{T_h - T_c}{T_h} \nonumber$$

$$\eta_{ef\,f} = 1 - \frac{T_c}{T_h} \label{8.6.6}$$

Equation \ref{8.6.6} is known as the Carnot efficiency. It provides a serious limitation on the efficiency of energy conversion devices which involve converting energy of a temperature difference to another form. The Carnot efficiency applies to thermoelectric devices, steam turbines, coal power plants, pyroelectric devices, and any other energy conversion device that convert a temperature difference into another form of energy. It does not, however, apply to photovoltaic or piezoelectric devices. If the hot side of a device is at the same temperature as the cold side, we cannot extract any energy. If the cold side of a device is at room temperature, then the efficiency cannot be 100%. The Carnot efficiency represents the best possible efficiency, not the actual efficiency of a particular device because it is likely that

other energy conversion processes occur too. We can extract more energy from a steam turbine with $T_h = 495$ K than $T_h = 295$ K. However, in both cases, the amount of energy we can extract is limited by the Carnot efficiency. Note that when using Equation \ref{8.6.6}, T_c and T_h must be specified on an absolute temperature scale, where $T = 0$ is absolute zero. In SI units, we use temperature in kelvins.

As an example, consider a device that converts a temperature difference into kinetic energy. The cold side of the device is at room temperature, $T_c = 300$ K. How hot must the hot side of the device be heated to so that the device achieves 40% efficiency?

$$\eta_{ef\,f} = 1 - \frac{T_c}{T_h} \nonumber$$

$$0.4 = 1 - \frac{300}{T_h} \nonumber$$

According to Equation \ref{8.6.6}, we find that $T_h = 500$ K.

As another example, suppose we want to convert a temperature differential to electrical energy using a thermoelectric device. Assume that the cold side of the device is at room temperature of $T_c = 72 ^{\circ}F$ and the hot side is at human body temperature of $T_h = 96 ^{\circ}F$. What is the best possible efficiency? First the temperatures must be converted from degrees Fahrenheit to kelvins. The resulting temperatures are $T_c = 295$ K and $T_h = 309$ K. Next, using Equation \ref{8.6.6}, we find the best possible efficiency is only 4.5%.

$$\eta_{ef\,f} = 1 - \frac{295}{309} = 0.045 \nonumber$$

As another example, assume that the temperature outside on a December day is $T_c = 20 ^{\circ}F$ and inside room temperature is $T_h = 72 ^{\circ}C$. What is the Carnot efficiency of a thermoelectric device operating at these temperatures? Again we begin by converting the temperatures to kelvins, $T_c = 266$ K and $T_h = 295$ K.

$$\eta_{ef\,f} = 1 - \frac{266}{295} = 0.098 \nonumber$$

Other Factors That Affect Efficiency

The efficiency of practical energy conversion devices is always lower than the Carnot efficiency because it is very unlikely that only a single energy conversion process occurs. All practical materials, even good conductors, have a finite resistance, so energy is converted to thermal energy as charges travel through the bulk of the device and through wires connected to it. Furthermore, heat flows through the device, so if a heater is connected to one side of a device, the other side will be at a higher temperature than the room it is in. For this reason, not all energy supplied by the heater can be converted to electricity.

As an example, consider a material with length $l = 1 mm = 10^{-3} m$ and cross sectional area $A = 1 mm^2 = 10^{-6} m^2$. Assume the material has a resistivity of $\rho = 10^{-5} \Omega \cdot m$ which is typical for a moderate conductor. Assume a current of $I = 3$ mA flows through the sample. How much power is converted to heat due to resistive heating? The electrical conductivity of the sample is $\sigma = \frac{1}{\rho} = 10^5 \frac{1}{\Omega \cdot m}$. The resistance of the device is given by $R = \frac{\rho l}{A}$. Power is

$$P=I^{2} R=I^{2} \frac{\rho l}{A}=\left(3 \cdot 10^{-3}\right)^{2} \frac{10^{-5} \cdot 10^{-3}}{10^{-6}}=9 \cdot 10^{-8}$$

\mathrm{W} \nonumber \]

While this amount of power may seem small, it is another factor which diminishes the efficiency of the device. Even if we convert energy from a temperature differential to electricity at the junction of the thermoelectric device, some resistive heating occurs. This heat is wasted in the sense that it isn't converted back to electricity.

The efficiency of most thermoelectric devices is less than 10% [5, p. 140] [117]. As seen by Equation \ref{8.6.6}, efficiency depends heavily on the temperatures \(T_c\) and \(T_h\), and efficiency can be increased by increasing \(T_h\). For many devices, the maximum temperature is limited by material considerations including the melting temperature.