The expanded form of the governing equation corresponding to the assumed type of loading is
\[
D \left( \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) + \bar{N} \frac{d^2 w}{dx^2} = 0
\]

The solution of the above linear partial differential equation with constant coefficient is sought as a product of two harmonic functions
\[
w(x, y) = \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b}
\]
where \(m\) and \(n\) are number of half waves in the longitudinal and transverse directions, respectively. The function \(w(x, y)\) satisfies the boundary condition for displacement. The bending moment \(M_n\)
\[
M_n = M_{xx} = D \left[ \kappa_{xx} + \nu \kappa_{yy} \right] = -D \left( \frac{m \pi}{a} \right)^2 \left[ \left( \frac{m}{a} \right)^2 + \nu \left( \frac{n}{b} \right)^2 \right] \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b}
\]
vanishes at \(x = 0\) and \(x = a\) edges. Also at \(y = 0\) and \(y = b\), \(M_n = M_{yy}\) is zero. Therefore the proposed function satisfy the simply supported boundary condition at all four edges. Substituting the function \(w(x, y)\) into the governing equation, one gets
\[
D \left( \frac{m \pi}{a} \right)^4 \left[ \left( \frac{m}{a} \right)^2 + \nu \left( \frac{n}{b} \right)^2 \right]^2 \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b} = 0
\]
The differential equation is satisfied for all values of \((x, y)\) if the coefficients satisfy
\[
\bar{N} = D \left( \frac{\pi a}{m} \right)^2 \left[ \left( \frac{m}{a} \right)^2 + \left( \frac{n}{b} \right)^2 \right]^2
\]
It is seen that the smallest value of $\bar{N}$ for all values of $a$, $b$, and $m$ is obtained if $n = 1$. This means that only one half wave will be formed in the direction perpendicular to the load application. Then, Equation \ref{11.12} can be put into a simple form

$$\bar{N}_c = k_c \frac{\pi^2 D}{b^2}$$

where the buckling coefficient $k_c$ is a function of both the plate aspect ratio $a/b$ and the wavelength parameter

$$k_c = \left( \frac{mb}{a} + \frac{a}{mb} \right)^2$$

The parameter $m$ is an integer and determines how many half waves will fit into the length of the plate. The aspect ratio $a/b$ is known, but the wavelength parameter is still unknown. Its value must be found by inspection, i.e., by plotting the buckling coefficient as a function of $a/b$ for subsequent values of the parameter $m$. This is shown in Figure (\ref{PageIndex{1}}).

For example, the buckling coefficient corresponding to the first five buckling modes corresponding to $a/b = 2$ are

<table>
<thead>
<tr>
<th>$m$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_c$</td>
<td>6.2</td>
<td>4</td>
<td>4.7</td>
<td>6.2</td>
<td>8.4</td>
</tr>
</tbody>
</table>

The lowest buckling load $k_c = 4$ occurs when there are two half waves along the length of the plate, $m = 2$. The line separating the safe, shaded area in Figure (\ref{PageIndex{2}}) and the unsafe while area defines uniquely the buckling coefficient for all combination of $a/b$ and $m$.

Consider now a long plate, $a \gg b$ for which the parameter $m$ can be treated as a continuous variable. In this case there is an analytical minimum of the buckling coefficient

$$\left[ \frac{dk_c}{dm} = 0 \quad \rightarrow \quad a = mb \right]$$
The above result means that the plate divides itself into an integer number of squares with alternating convex and concave dimples.

What happens when the rectangular plate shown in Figure (10.1.1) is restricted from lateral expansion

\[ u_y(y = 0) = u_y(y = b) = 0 \]

![Figure 10.1.1: Constrained compression of the plate.](image)

With no strain in the y-direction, \( \epsilon_{yy} = 0 \), the constitutive equations (11.6) reduces to

\[ N_{xx} = C \epsilon_{xx} \]
\[ N_{yy} = C \nu \epsilon_{xx} \]

This means that a reaction force \( N_{yy} = \nu N_{xx} \) develops in the transverse direction. The buckled shape of the plate is the same and the solution, Equation (??) still holds but the new expression for the buckling coefficient is

\[ k_c = \frac{\left[ \left( \frac{mb}{a} \right)^2 + n^2 \right]^2}{\left( \frac{mb}{a} \right)^2 + \nu n^2} \]

The least value of the buckling coefficient can be found by inspection. Taking again as an example \( a/b = 2 \), the values of the buckling coefficient corresponding to the nine first buckling modes are

<table>
<thead>
<tr>
<th>( n )</th>
<th>( m )</th>
<th>( \frac{mb}{a} )</th>
<th>( \nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10/7</td>
<td>3</td>
<td>4.09</td>
</tr>
<tr>
<td>2</td>
<td>3.8</td>
<td>10.7</td>
<td>10.9</td>
</tr>
<tr>
<td>3</td>
<td>26</td>
<td>25</td>
<td>24.1</td>
</tr>
</tbody>
</table>

The lowest value of the buckling coefficient \( k_c = 3 \) corresponds to two half-waves in the loading direction and one half wave in the transverse direction. It is seen that restricting the in-plane deformation does not change the buckling mode but reduces the buckling load by a factor of \( 3/4 \). The reaction compressive force makes the plate to buckle more easily. This example underscores the importance of properly defining the boundary conditions not only in the out-of-plane direction but also in the in-plane directions.

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