If the stresses within a beam exceed the elastic limit, then plastic deformation will occur. This can dramatically change
the behaviour. Consider a material exhibiting elastic - perfectly plastic behaviour (i.e. no work-hardening), as shown
below.
Stress-strain curve for an elastic-perfectly plastic material.

Stress and strain distributions before and after applying the moment are shown below. In the outer regions of the beam, the stress will be capped at $\sigma_Y$, although the strain will continue to increase linearly with distance from the neutral axis, as in the elastic case. The curvature (strain gradient), $\kappa$, induced by a given moment, $M$, will now be greater, since this increase will be required in order to bring the internal moment back up to the level of the applied moment - i.e. bending will increase.

Distributions of stress and strain within a beam before and after application of a moment sufficiently large to cause plastic deformation

A further difference is observed on removal of the applied moment, since the beam will now retain a residual curvature, $\kappa_{res}$, as a result of the plastic deformation. This is due to the presence of residual stresses. The residual curvature can be calculated, using the fact that the beam is subject to no applied force. It follows that the residual stress distribution must satisfy a force balance, so that

$$\int_{y=0}^{y_s} \sigma(y) \, dy = 0$$

which is equivalent to the shaded areas in the diagram being equal. Since the change in stress (at any value of $y$) on removing the applied moment is given by the change in strain at that depth times the modulus (eg $= E \Delta \varepsilon$ at $y = y_s$ - see diagram), these equations allow the residual stress distribution to be established. The following expressions can be obtained (click here) for the thickness of the elastic core, the residual curvature, the surface residual stress and the residual stress at the limit of the elastic core.
\[ y_{\text{e}} = \frac{\sigma_{Y}}{E \kappa} \]

\[ \kappa_{\text{res}} = \kappa \left(1 - \frac{y_{\text{e}}}{y_{\text{s}}} \right)^2 \]

\[ \sigma_{\text{s, res}} = \sigma_{Y} - E y_{\text{s}} \left(\kappa - \kappa_{\text{res}} \right) \]

\[ \sigma_{\text{e, res}} = \sigma_{Y} - E y_{\text{e}} \left(\kappa - \kappa_{\text{res}} \right) \]

Of course, the picture may in practice be complicated by work hardening, more complex sectional geometries, non-prismatic beams etc, but the same principles still apply. Incidentally, it may be noted that, in addition to the force balance, the residual stress distribution in an unloaded beam must also satisfy a moment balance, so that

\[ \int_{y=0}^{y_{\text{s}}} \sigma(y) y \, dy = 0 \]

However, the symmetry of the tensile and compressive sides of the beam ensures that this condition is satisfied, so it is not involved in the solution procedure in this case. In other cases, however, in which the neutral axis is not a plane of symmetry, this condition may also need to be invoked in order to find the solution.

The plastic deformation behaviour of a prismatic beam, with a symmetrical, rectangular section, made of a metal exhibiting no work hardening, can be explored using the plastic version of the beam bending simulation presented in an earlier section.