4.4: Disturbance Rejection

System Response to Disturbance Inputs

Disturbances are unwanted signals entering into a feedback control system. A disturbance may act at the input or output of the plant. Here we consider the effect of input disturbance.

To characterize the effect of a disturbance input on the feedback control system, we consider the modified block diagram (Figure 4.4.1) that includes a disturbance input.

Figure \(\PageIndex{1}\): A feedback control system with reference and disturbance inputs.

Let \(r(t)\) denote a reference, and \(d(t)\) a disturbance input; then the system output is expressed in the Laplace domain as:

\[
y(s) = \frac{KG(s)}{1+KGH(s)} r(s) + \frac{G(s)}{1+KGH(s)} d(s)
\]

Assuming unity-gain feedback configuration \((H(s)=1)\), the tracking error, \(e(s)\), is computed as:

\[
e(s) = \frac{1}{1+KG(s)} r(s) - \frac{G(s)}{1+KG(s)} d(s)
\]

By using the FVT, the steady-state error is expressed as:
\[ e(\infty) = \frac{1}{1 + K_p} \cdot r(\infty) - \frac{G(0)}{1 + K_p} \cdot d(\infty) \]

where \(K_p\) is the position error constant.

A large loop gain (large \(K_p\)) reduces steady-state error in the presence of both reference and disturbance inputs. A large controller gain, \(K\), can be used to increase \(K_p\), however, a large \(K\) would generate a large magnitude input signal to the plant, which may cause saturation in the actuator devices (amplifiers, mechanical actuators, etc.).

### Simultaneous Tracking and Disturbance Rejection

To analyze the control requirements for simultaneous tracking and disturbance rejection, we consider a unity-gain feedback control system (\(H(s) = 1\)).

Let \(G(s) = \frac{n(s)}{d(s)}\) represent the plant and \(K(s) = \frac{n_C(s)}{d_C(s)}\) represent the controller; then, the output in the presence of reference and disturbance inputs is given as:

\[
y(s) = \frac{d(s)d_c(s)}{n(s)n_c(s)+d(s)d_c(s)}r(s) - \frac{n(s)d_c(s)}{n(s)n_c(s)+d(s)d_c(s)}d(s)
\]

The characteristic polynomial is given as: \(\Delta(s) = n(s)n_c(s)+d(s)d_c(s)\).

The requirements for asymptotic tracking and disturbance rejection are stated as follows:

**Asymptotic tracking.** For asymptotic tracking, \(d(s)d_c(s)\) should contain any unstable poles of \(r(s)\). For example, an integrator in the feedback loop ensures zero steady-state error to a constant reference input.

**Disturbance Rejection.** For disturbance rejection, \(n(s)d_c(s)\) should contain any unstable poles of \(d(s)\). For example, a notch filter centered at \(60\ Hz\) removes power line noise from the measured signal.

Example \(\PageIndex{1}\)

A small DC motor has the following component values: \(R = 1\; \Omega\), \(\Omega = 10\; \text{rad/s}\), \(J = 0.01\; \text{kgm}^2\), \(b = 0.1\; \text{Nms}\), \(k_t = k_b = 0.05\).

The DC motor transfer function is given as: \(G(s) = \frac{500}{s^2 + 110s + 1025}\).

Since \(G(0) \cong 0.5\), with a static controller, the position error constant is: \(K_p = 0.5K\). The steady-state error in the presence of reference and disturbance inputs is given as:

\[ e(\infty) = \frac{1}{1 + 0.5K} \cdot r(\infty) - \frac{0.5}{1 + 0.5K} \cdot d(\infty) \]

We may choose a large \(K\) to reduce the steady-state tracking error as well as improve the disturbance rejection. A large value of \(K\), however, reduces system damping and results in an oscillatory response of the DC motor.

For a static controller, the closed-loop characteristic polynomial is given as: \(\Delta(s,K) = s^2 + 110s + 1025 + 500K\).

The resulting damping ratio is: \(\zeta = \frac{55}{\sqrt{1025 + 500K}}\).
In order to limit the damping to, say $\zeta \le 0.6$, the controller gain is limited to: $K \le 14.75$. We may choose, for example, $K=14$, which results in the following steady-state error to reference and disturbance inputs: $e(\infty) = \frac{1}{8} r(\infty) - \frac{1}{16} d(\infty)$.\)