1.0: Prelude to Mathematical Models of Physical Systems

This chapter describes the process of obtaining the mathematical description of a dynamic system, i.e., a system whose behavior changes over time. The system is assumed to be assembled from components. The system model is based on the physical laws that govern the behavior of various system components.

Physical systems of interest to engineers include, for example, electrical, mechanical, electromechanical, thermal, and fluid systems. By using lumped parameter assumption, their behavior is mathematically described in terms of ordinary differential equation (ODE) models. These equations are nonlinear, in general, but can be linearized about an operating point for analysis and design purposes.

Models of interconnected components are assembled from individual component descriptions. The components in electrical systems include resistors, capacitors, and inductors. The components used in mechanical systems include inertial masses, springs, and dampers (or friction elements). For thermal systems, these include thermal capacitance and thermal resistance. For hydraulic and fluid systems, these include reservoir capacity and flow resistance.

In certain physical systems, properties (or entities) flow in and out of a system boundary, e.g., a hydraulic reservoir, or thermal chamber. The dynamics of such systems is described by conservation laws and/or balance equations. In particular, let Q represent an accumulated property, \( q_{\text{in}} \) and \( q_{\text{out}} \) represent the inflow and outflow rates, then the model is described as:

\[
\frac{dQ}{dt} = q_{\text{in}} - q_{\text{out}} + g - c
\]

The Laplace transform converts a linear differential equation into an algebraic equation, which can be manipulated to obtain an input-output description described as a transfer function. The transfer function forms the basis of analysis and design of control systems using conventional methods. In contrast, the modern control theory is established on time-domain analysis involving the state equations, that describe system behavior as time derivatives of a set of state
variables.

Linearization of nonlinear models is accomplished using Taylor series expansion about a critical point, where the linear behavior is restricted to the neighborhood of the critical point. The linear systems theory is well-established and serves as the basic tool for controller design.