5.2: Hydraulic Transport of Sand-Shell Mixtures in Relation with the LDV

5.2.1 Introduction

This chapter is based on Ramsdell & Miedema (2010), Ramsdell et al. (2011) and Miedema & Ramsdell (2011). When considering pumping shells through a pipeline we have to consider that the shells are not spherical, but more disc shaped. When shells settle they will settle like leaves where the biggest cross section is exposed to the drag. But when they settle, they will settle in the same orientation, flat on the sediment, so the side of the shells is exposed to the horizontal flow in the pipeline. Since the side cross section is much smaller than the horizontal cross section, a much higher velocity is required to make them erode and go back into suspension. The settling velocity is much smaller because of the large area of the cross section.

Now normally pipeline resistance is calculated based on the settling velocity, where the resistance is proportional to the settling velocity of the grains. The Limit Deposit Velocity (LDV) is also proportional to the settling velocity. Since shells have a much lower settling velocity than sand grains with the same weight and much lower than sand grains with the same sieve diameter, one would expect a much lower resistance and a much lower critical velocity, matching the lower settling velocity. Now this is only partly true. As long as the shells are in suspension, on average they want to stay in suspension because of the low settling velocity. But as stated before, settling and erosion are stochastic processes because of the turbulent character of the flow in the pipeline. Since we operate at Reynolds numbers above 1 million the flow is always turbulent, meaning that eddies and vortices occur stochastically making the particles in the flow move up and down, resulting in some particles hitting the bottom of the pipe. Normally these particles will be picked up in the flow because of erosion, so there exists equilibrium between sedimentation and erosion, resulting in not having a bed at the bottom of the pipeline. In fact the capacity of the flow to erode is bigger than the sedimentation. If the line speed decreases, the shear velocity at the bottom of the pipe also decreases and less particles will be eroded, so the erosion capacity is decreasing. Now this does not matter as long as the erosion capacity is bigger than the sedimentation there
will not be sediment at the bottom of the pipeline. As soon as the line speed decreases so much that the erosion capacity (erosion flux) is smaller than the sedimentation flux, not all the particles will be eroded, resulting in a bed to be formed at the bottom of the pipe. Having a bed at the bottom of the pipe also means that the cross section of the pipe decreases and the actual flow velocity above the bed increases. This will result in a new equilibrium between sedimentation flux and erosion flux for each bed height.

So from the moment there is a bed, decreasing the flow will result in an almost constant flow velocity above the bed, resulting in equilibrium between erosion and sedimentation. This equilibrium however is sensitive for changes in the line speed and in the mixture density. Increasing the line speed will reduce the bed height; a decrease will increase the bed height. Having a small bed does not really matter, but a thick bed makes the system vulnerable for plugging the pipeline. The LDV in most models is chosen in such a way that a thin bed is allowed. Now for the shells, as said before, there will always be some shells that will reach the bottom of the pipe due to the combination of settling velocity and turbulence. Once these shells are on top of the sediment they are hard to remove by erosion, because they lay flat on the surface and have a small cross section that is exposed to the flow compared with the weight of the shell. So although their settling velocity is much lower than equivalent sand particles, the erosion velocity is much higher. If we look at the beach in an area with many shells, we can always see the shells on top of the sand, covering the sand. In fact the shells are shielding the sand from erosion, because they are hard to erode. The bigger shells will also shield the smaller pieces, because the smaller pieces settle faster.

Compare this with leaves falling from a tree, the bigger leaves, although heavier, will fall slower, because they are exposed to higher drag. The same process will happen in the pipeline. Shells settle slower than sand grains, so they will be on top of the bed (if there is a bed), just like on the beach. Since they are hard to erode, in fact they protect the bed from being eroded, even if the line speed is increased. But there will always be velocities above the bed that will make the shells erode. Now the question is how we can quantify this behavior in order to get control over it. We have to distinguish between sedimentation and erosion. First of all assume shells are disc shaped with a diameter $d$ and a thickness of $\alpha \cdot d$ and let’s take $\alpha = 0.1$ this gives a cross section for the terminal settling velocity of $\frac{\pi}{4} d^2$, a volume of $\frac{\pi}{40} d^3$ and a cross section for erosion of $\frac{d^2}{10}$. Two processes have to be analyzed to determine the effect of shells on the critical velocity, the sedimentation process and the erosion process.

### 5.2.2 The Drag Coefficient

The drag coefficient $C_D$ depends upon the Reynolds number according to Turton & Levenspiel (1986), which is a 5 parameter fit function to the data:

$$C_D = \frac{24}{Re_p} \cdot \left(1 + 0.173 \cdot Re_p^{0.657}\right) + \frac{0.413}{1 + 16300 \cdot Re_p^{-1.09}}$$

It must be noted here that in general the drag coefficients are determined based on the terminal settling velocity of the particles. Wu & Wang (2006) recently gave an overview of drag coefficients and terminal settling velocities for different particle Corey shape factors. The result of their research is reflected in Figure 5.2-1. Figure 5.2-1 shows the drag coefficients as a function of the Reynolds number and as a function of the Corey shape factor. Figure 5.2-1: Drag coefficient as a function of the particle shape (Wu & Wang, 2006).
For shells settling the Corey shape factor is very small, like 0.1, resulting in high drag coefficients. According to Figure 5.2-1 the drag coefficient should be like:

\[
C_D = \frac{32}{Re_p} + 2 \text{ up to } C_D = \frac{36}{Re_p} + 3
\]

For shells lying flat on the bed, the drag coefficient will be similar to the drag coefficient of a streamlined half body (0.09), which is much smaller than the drag coefficient for settling (3). So there is a large asymmetry between the settling process and the erosion process of shells, while for more or less spherical sand particles the drag coefficient is considered to be the same in each direction.

### 5.2.3 Non-Uniform Particle Size Distributions

In the model for uniform particle distributions, the roughness \( k_s \) was chosen equal to the particle diameter \( d \), but in the case of non-uniform particle distributions, the particle diameter \( d \) is a factor \( d^+ \) times the roughness \( k_s \), according to:

\[
d^+ = \frac{d}{k_s}
\]

The roughness \( k_s \) should be chosen equal to some characteristic diameter related to the non-uniform particle distribution, for example the \( d_{50} \).
5.2.4 Laminar Region

For the laminar region (the viscous sub layer) the velocity profile of Reichardt (1951) is chosen. This velocity profile
gives a smooth transition going from the viscous sub layer to the smooth turbulent layer.

\[
\frac{u_{\text{top}}^+}{u_*} = \frac{\ln \left( 1 + \kappa \cdot y_{\text{top}}^+ \right)}{\kappa} - \frac{\ln (1/9) + \ln (\kappa)}{\kappa} \cdot \left( 1 - e^{-\frac{y_{\text{top}}^+}{11.6}} - \frac{y_{\text{top}}^+}{11.6} e^{-0.33 \cdot y_{\text{top}}^+} \right) \approx y_{\text{top}}^+
\]

For small values of the boundary Reynolds number and thus the height of a particle, the velocity profile can be made
linear to:

\[
\frac{u_{\text{top}}^+}{d^+ \cdot E} = \frac{u_{\text{eff}}^+}{d^+ \cdot E} = \frac{k_s^+}{d^+ \cdot E} = \frac{k_s^+}{y_{\text{top}}^+}
\]

Adding the effective turbulent velocity to the time averaged velocity, gives for the velocity function \( \alpha_{\text{Lam}} \):

\[
\alpha_{\text{Lam}} = y_{\text{top}}^+ + u_{\text{eff}}^+ \left( y_{\text{top}}^+ \right)
\]

5.2.5 Turbulent Region

Particles that extend much higher into the flow will be subject to the turbulent velocity profile. This turbulent velocity
profile can be the result of either a smooth boundary or a rough boundary. Normally it is assumed that for boundary
Reynolds numbers less than 5 a smooth boundary exists, while for boundary Reynolds numbers larger than 70 a rough
boundary exists. In between in the transition zone the probability of having a smooth boundary is:

\[
P = e^{-0.95 \cdot \frac{R e_*}{11.6}} = e^{-0.95 \cdot \frac{k_s^+}{11.6}}
\]

This probability is not influenced by the diameter of individual particles, only by the roughness \( k_s \) which is determined by
the non-uniform particle distribution as a whole. This gives for the velocity function \( \alpha_{\text{Turb}} \):

\[
\alpha_{\text{Turb}} = \frac{1}{\kappa} \cdot \ln \left( 95 \cdot \frac{y_{\text{top}}^+}{\delta_{\text{v}}^+} + 1 \right) \cdot P + \frac{1}{\kappa} \cdot \ln \left( 30 \cdot \frac{y_{\text{top}}^+}{k_s^+} + 1 \right) \cdot (1-P)
\]

The velocity profile function has been modified slightly by adding 1 to the argument of the logarithm. Effectively this
means that the velocity profile starts \( y_0 \) lower, meaning that the virtual bed level is chosen \( y_0 \) lower for the turbulent
region. This does not have much effect on large exposure levels (just a few percent), but it does on exposure levels of
0.1 and 0.2. Not applying this would result in to high (not realistic) shear stresses at very low exposure levels.
5.2.6 The Exposure Level

Effectively, the exposure level $E$ is represented in the equations (5.1-34), (5.1-38) and (5.1-42) for the Shields parameter by means of the velocity distribution according to equations (5.2-6) and (5.2-8) and the sliding friction coefficient $\mu sf$ or the pivot angle $\psi$. A particle with a diameter bigger than the roughness $k_s$ will be exposed to higher velocities, while a smaller particle will be exposed to lower velocities. So it is important to find a relation between the non-dimensional particle diameter $d^+$ and the exposure level $E$.

5.2.7 The Angle of Repose & the Friction Coefficient

Miller & Byrne (1966) found the following relation between the pivot angle $\psi$ and the non-dimensional particle diameter $d^+$, with $c_0=61.5^\circ$ for natural sand, $c_0=70^\circ$ for crushed quartzite and $c_0=50^\circ$ for glass spheres.

\[
\psi = c_0 \cdot (d^+)^{-0.3}
\]

Wiberg & Smith (1987A) re-analyzed the data of Miller & Byrne (1966) and fitted the following equation:

\[
\psi = \cos^{-1}\left(\frac{d^+ + z^*}{d^+ + 1}\right)
\]

The average level of the bottom of the almost moving grain $z^*$ depends on the particle sphericity and roundness. The best agreement is found for natural sand with $z^*=-0.045$, for crushed quartzite with $z^*=-0.320$ and for glass spheres with $z^*=-0.285$. Wiberg & Smith (1987A) used for natural sand with $z^*=-0.020$, for crushed quartzite with $z^*=-0.160$ and for glass spheres with $z^*=14$. The values found here are roughly 2 times the values as published by Wiberg & Smith (1987A). It is obvious that equation (5.2-10) underestimates the angle of repose for $d+$ values smaller than 1.

5.2.8 The Equal Mobility Criterion

Now two different cases have to be distinguished. Particles with a certain diameter can lie on a bed with a different roughness diameter. The bed roughness diameter may be larger or smaller than the particle diameter. Figure 5.2-3 shows the Shields curves for this case (which are different from the graph as published by Wiberg & Smith (1987A)), combined with the data of Fisher et al. (1983), and based on the velocity distributions for non-uniform particle size distributions. Fisher et al. carried out experiments used to extend the application of the Shields entrainment function to both organic and inorganic sediments over passing a bed composed of particles of different size. Figure 5.2-3 shows a good correlation between the theoretical curves and the data, especially for the cases where the particles considered are bigger than the roughness diameter ($d/k_s>1$). It should be noted that most of the experiments were carried out in the transition zone and in the turbulent regime. Figure 5.2-3 is very important for determining the effect of shells on a bed, because with this figure we can determine the critical Shields parameter of a particle with a certain diameter, lying on a bed with a roughness of a different diameter. In the case of the shells the bed roughness diameter will be much smaller than the shell diameter (dimensions). To interpret Figure 5.2-3 one should first determine the bed roughness diameter and the roughness Reynolds number and take the vertical through this roughness Reynolds number (also called the boundary Reynolds number). Now determine the ratio $d/k_s$ and read the Shields parameter from the graph. From this it appears that the bigger this ratio, the smaller the Shields value found. This is caused by the fact that the Shields...
parameter contains a division by the particle diameter, while the boundary shear stress is only influenced slightly by the changed velocity distribution. Egiazaroff (1965) was one of the first to investigate non-uniform particle size distributions with respect to initiation of motion. He defined a hiding factor or exposure factor as a multiplication factor according to:

\[
\theta_{cr,i} = \theta_{cr,d50} \cdot \left(\frac{\log(19)}{\log\left(19 \cdot \frac{d_i}{d_{50}}\right)}\right)^2
\]

The tendency following from this equation is the same as in Figure 5.2-3, the bigger the particle, the smaller the Shields value, while in equation (5.2-11) the \(d_{50}\) is taken equation to the roughness diameter \(k_s\). The equal mobility criterion is the criterion stating that all the particles in the top layer of the bed start moving at the same bed shear stress, which matches the conclusion of Miedema (2010) that sliding is the main mechanism of entrainment of particles. Figure 5.2-4 shows that the results of the experiments are close to the equal mobility criterion, although not 100%, and the results from coarse sand from the theory as shown in Figure 5.2-3, matches the equal mobility criterion up to a ratio of around 10. Since shells on sand have a \(d/k_s\) ratio bigger than 1, the equal mobility criterion will be used for the interpretation of the shell experiments as also shown in Figure 5.2-3.

5.2.9 Shells

Dey (2003) has presented a model to determine the critical shear stress for the incipient motion of bivalve shells on a horizontal sand bed, under a unidirectional flow of water. Hydrodynamic forces on a solitary bivalve shell, resting over a sand bed, are analyzed for the condition of incipient motion including the effect of turbulent fluctuations. Three types of bivalve shells, namely Coquina Clam, Cross-barred Chione and Ponderous Ark, were tested experimentally for the condition of incipient motion. The shape parameter of bivalve shells is defined appropriately.

Although the model for determining the Shields parameter of shells is given, the experiments of Dey (2003) were not translated into Shields parameters. It is interesting however to quantify these experiments into Shields parameters and to see how this relates to the corresponding Shields parameters of sand grains. In fact, if the average drag coefficient of the shells is known, the shear stress and thus the friction velocity, required for incipient motion, is known, the flow velocity required to erode the shells can be determined. Figure 5.2-5 and Figure 5.2-6 give an impression of the shells used in the experiments of Dey (2003). From Figure 5.2-5 it is clear that the shape of the shells match the shape of a streamlined half body lying on a surface and thus a drag coefficient is expected of about 0.1, while sand grains have a drag coefficient of about 0.45 at very high Reynolds numbers in a full turbulent flow. The case considered here is the case of a full turbulent flow, since we try to relate the incipient motion of shells to the critical velocity.

Equation (5.1-34) shows the importance of the drag coefficient in the calculation of the incipient motion, while the lift coefficient is often related to the drag coefficient. Whether the latter is true for shells is the question. For sand grains at high Reynolds numbers of then the lift coefficient is chosen to be 0.85 times the drag coefficient or at least a factor between 0.5 and 1, shells are aerodynamically shaped and also asymmetrical. There will be a big difference in the lift coefficient of shells lying on the bed, between convex upwards and convex downwards. A convex upwards shell is like the streamlined half body with a small drag coefficient. A convex downwards shell obviously is easy to catch the flow and start to move, because the drag coefficient is larger and most probably, the lift coefficient is much larger. So it will be the convex upwards shells that armor the bed or the beach.
Now the question is, what the drag coefficient would be, based on the experiments of Dey (2003). Figure 5.2-7 shows the Shields parameters for the three types of shells lying convex upwards on the bed with two types of sand, a \( d_{50} = 0.8 \) mm and a \( d_{50} = 0.3 \) mm, also the average values are shown. For the determination of the Shields values, the definition of the Shields parameter has to be used more strictly. Often a definition is used where the Shields parameter equals the ratio between the shear force and the normal force on the grain, resulting in a denominator with the particles diameter.

![Non-uniform particle distributions](https://eng.libretexts.org/Bookshelves/Civil_Engineering/Book%3A_Slurry_Transport_(Miedema)/05%3A_Initiation_of_Motion…)

Figure 5.2-3: Non-uniform particle distributions.

![Critical bed shear stress of individual size fractions in a mixture as a function of grain diameter (modified after van Rijn (2006) and Wilcock (1993)).](https://eng.libretexts.org/Bookshelves/Civil_Engineering/Book%3A_Slurry_Transport_(Miedema)/05%3A_Initiation_of_Motion…)

Figure 5.2-4: Critical bed shear stress of individual size fractions in a mixture as a function of grain diameter (modified after van Rijn (2006) and Wilcock (1993)).

![Shape of bivalve shell (Dey (2003)).](https://eng.libretexts.org/Bookshelves/Civil_Engineering/Book%3A_Slurry_Transport_(Miedema)/05%3A_Initiation_of_Motion…)

Figure 5.2-5: Shape of bivalve shell (Dey (2003)).
More strictly, the Shields parameter is the shear stress divided by the normal stress and in the case of shells; the normal stress depends on the average thickness of the shell and not the size of the shell. Using this definition, results in useful Shields values. Since convex upwards is important for the critical velocity analysis, this case will be analyzed and discussed. It is clear however from these figures that the convex downwards case results in much smaller Shields values than the convex upwards case as was expected. Smaller Shields values in this respect means smaller shear stresses and thus smaller velocities above the bed causing erosion. In other words, convex downwards shells erode much easier than convex upwards.

Although the resulting Shields values seem to be rather stochastic, it is clear that the mean values of the Chione and the Coquina are close to the Shields curve for \( \frac{d}{k_s}=1 \). The values for the Ponderous Ark are close to the Shields curve for \( \frac{d}{k_s}=3 \). In other words, the Ponderous Ark shells are easier to erode than the Chione and the Coquina shells. Looking at the shells in Figure 5.2-6 we can see that the Ponderous Ark shells have ripples on the outside and will thus be subject to a higher drag. On the other hand, the Ponderous Ark shells have an average thickness of 2.69 mm (1.95-3.98 mm) as used in the equation of the Shields parameter, while the Coquina clam has a thickness of 1.6 mm (0.73-3.57 mm) and the Chione 1.13 mm (0.53-2.09 mm). This also explains part of the smaller Shields values of the Ponderous Ark. The average results of the tests are shown in the following table.

Figure 5.2-6: Selected samples of bivalve shells (Dey (2003)).

Figure 5.2-7: Shells convex upward.
Figure 5.2-8: The critical shear stresses of the shells compared with sand.

Table 5.2-1: Average Shields values.

<table>
<thead>
<tr>
<th>Shell Type</th>
<th>$d_{50}$ (mm)</th>
<th>$d_{50}=0.8$ mm</th>
<th>$d_{50}=0.3$ mm</th>
<th>$d_{50}=0.8$ mm</th>
<th>$d_{50}=0.3$ mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coquina Clam</td>
<td>19.78</td>
<td>0.0277</td>
<td>6.71</td>
<td>0.0225</td>
<td>1.60</td>
</tr>
<tr>
<td>Cross Barred Chione</td>
<td>17.51</td>
<td>0.0378</td>
<td>6.24</td>
<td>0.0333</td>
<td>1.13</td>
</tr>
<tr>
<td>Ponderous Ark</td>
<td>18.46</td>
<td>0.0129</td>
<td>5.76</td>
<td>0.0086</td>
<td>2.69</td>
</tr>
</tbody>
</table>

A closer look at the data, based on this table, shows the following. For the shells on the 0.8 mm sand the $d_{50}$ values...
vary from 1.41-3.36. The average Shields values found do not match the corresponding curves, but lead to slightly lower \(\frac{d}{ks}\) values. For example, the Cross Barred Chione had a Shields value of 0.0378, but based on the \(\frac{d}{ks}\) value of 1.41, a Shields value of about 0.02 would be expected, a ratio of 1.89. The Coquina Clam had an average Shields value of 0.0277, but based on the \(\frac{d}{ks}\) value of 2.00 a Shields value of about 0.015 would be expected, a ratio of 1.84. The Ponderous Ark had an average Shields value of 0.0129, but based on the \(\frac{d}{ks}\) value of 3.36 a Shields value of about 0.008 would be expected, a ratio of 1.61. For the 0.3 mm sand the average ratio is about 5.5. In other words, the shells require larger Shields values than corresponding sand grains. This effect is larger in the case of shells on a bed with finer sand particles. The exact ratios depend on the type of shells.

### 5.2.10 The Limit Deposit Velocity

A familiar phenomenon in the transport of sand slurries is the LSDV (Limit of Stationary Deposit Velocity), the velocity at which the mixture forms a stationary bed in the pipeline. As the velocity increases from the LSDV, the bed starts to slide along the bottom of the pipe. As the velocity increases further the bed begins to erode with the particles either rolling or saltating along the top of the bed, or fully suspended in the fluid, the LDV where all particles are in suspension.

A related concept is that of the minimum friction velocity, \(V_{\text{imin}}\), at which the friction in the pipeline is minimized. At low concentrations the \(V_{\text{imin}}\) may be equal to or just above the LDV, but as concentration increases the LDV starts to decrease while the \(V_{\text{imin}}\) continues to rise. In operational terms, the \(V_{\text{imin}}\) represents a point of instability, so we generally try to design our pumping systems to maintain sufficiently high velocities that the system velocity never falls below (or close to) \(V_{\text{imin}}\) during the operational cycle.

Implicit in most models of slurry transport is the idea that the system can transition smoothly in both directions along the system resistance curves. So if the dredge operator inadvertently feeds too high of a concentration, dropping the velocity close to the minimum friction or even the LDV, he can recover by slowly lowering the mixture concentration, which in turn lowers the density in the pipeline and allows the velocity to recover. Alternatively the operator can increase the pressure by turning up the pumps to raise the velocity. In a sand-sized material this works because the critical and minimum friction velocities are fairly stable, so raising the pumping velocity or lowering the concentration will be enough to start the bed sliding, then erode the bed and return to stable operation.

With a sand-shell mixture, as described above, the LDV and minimum friction velocities become time-dependent parameters. The stochastic nature of the process means that some fraction of the shells will fall to the bottom of the pipe. The asymmetry between deposition and erosion velocity means that these shells will stay on the bottom, forming a bed that grows over time, increasing the critical velocity and minimum friction velocity. Unless the system is operated with very high margins of velocity, the new LDV and \(V_{\text{imin}}\) eventually fall within the operating range of the system, leading to flow instability and possible plugging.

Now, how to combine this LDV with the erosion behavior of shells. As mentioned above, there are different models in literature for the LDV and there is also a difference between the LDV and the minimum friction velocity. However, whatever model is chosen, the real LDV is the result of an equilibrium of erosion and deposition resulting in a stationary bed. This equilibrium depends on the particle size distribution, the slurry density and the flow velocity. At very low concentrations it is often assumed that the LDV is zero, but based on the theory of incipient motion, there is always a certain minimum velocity required to erode an existing bed.
There are two ways to look at this problem, we can compare the Shields values of the shells with the Shields values of 
sand particles with a diameter equal to the thickness of the shells, resulting in the factors as mentioned in the previous 
paragraph or we compare the shear stresses occurring to erode the shells with the shear stresses required for the sand 
beds used. The latter seems more appropriate because the shear stresses are directly related to the average velocity 
above the bed with the following relation:

\[
\frac{\lambda_1}{8} \cdot \rho_1 \cdot U^2
\]

Where the left hand side equals the bed shear stress, $\lambda_1$ the friction coefficient following from the Moody diagram and $U$ 
the average flow velocity above the bed. The average shear stresses are shown in Table 5.2-2.

Table 5.2-2: Average shear stresses.

<table>
<thead>
<tr>
<th></th>
<th>$d_{50}=0.8$ mm</th>
<th>$d_{50}=0.3$ mm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Re^*$</td>
<td>$\rho_1 u^2$</td>
</tr>
<tr>
<td>Coquina Clam</td>
<td>19.78</td>
<td>0.0277</td>
</tr>
<tr>
<td>Cross Barred</td>
<td>17.51</td>
<td>0.0378</td>
</tr>
<tr>
<td>Chione</td>
<td>18.46</td>
<td>0.0129</td>
</tr>
</tbody>
</table>

The Shields values for both sands are about 0.035, resulting in shear stresses of 0.45 Pa for the 0.8 mm sand and 0.17 
Pa for the 0.3 mm sand. The ratios between the shear stresses required eroding the shells and the shear stresses 
required to erode the beds are also shown in Table 5.2-2. For the shells laying convex upwards on the 0.8 mm sand bed 
these ratio’s vary from 1.24-1.60, while this is a range from 2.18-3.41 for the 0.3 mm sand bed. These results make 
sense, the shear stress required for incipient motion of the shells does not change much because of the sand bed, 
although there will be some reduction for sand beds of smaller particles due to the influence of the bed roughness on the 
velocity profile according to equation (5.2-4). Smaller sand particles with a smaller roughness allow a faster 
development of the velocity profile and thus a bigger drag force on the shells at the same shear stress.

The main influence on the ratios is the size of the sand particles, because smaller particles require a smaller shear 
stress for the initiation of motion. This is also known from the different models for the LDV, the finer the sand grains, the 
smaller the critical velocity. In order words, the smaller the velocity to bring the particles in a bed back into suspension. It 
also makes sense that the ratio between shell erosion shear stress and sand erosion shear stress will approach 1 if the 
sand particles will have a size matching the thickness of the shells and even may become smaller than 1 if the sand 
particles are bigger than the shells.

Since the velocities are squared in the shear stress equation, the square root of the ratios has to be taken to get the
ratios between velocities. This leads to velocity ratio’s from 1.11-1.26 for the 0.8 mm sand and ratio’s from 1.48-1.89 for the 0.3 mm sand. Translating this to the LDV, can be carried out under the assumption that the LDV is proportional to the average flow velocity resulting in incipient motion. Although the LDV results from an equilibrium between erosion and deposition of particles and thus is more complicated, the here derived ratios can be used as a first attempt to determine the critical velocities for a sand bed covered with convex upwards shells.

For the coarser sands (around 0.8 mm) this will increase the critical velocity by 11%-26%, while this increase is 48%-89% for the finer 0.3 mm sand. Even finer sands will have a bigger increase, while coarser sands will have a smaller increase. As stated, the shear stress required to erode the shells is almost constant, but decreasing a little bit with decreasing sand particle diameters, an almost constant critical velocity for the shells is expected. From the measurements it is also clear, that very smooth shells (Coquina Clam and Cross Barred Chione) are harder to erode and will have a higher critical velocity than the rough shells (Ponderous Ark).

5.2.11 Conclusions and Discussion

The LDV for the hydraulic transport of a sand-water mixture depends on a number of physical processes and material properties. The LDV is the result of equilibrium between the deposition of sand particles and the erosion of sand particles. The deposition of sand particles depends on the settling velocity, including the phenomenon of hindered settling as described in this paper. The erosion or incipient motion of particles depends on equilibrium of driving forces, like the drag force, and frictional forces on the particles at the top of the bed. This results in the so called friction velocity and bottom shear stress. Particles are also subject to lift forces and so called Magnus forces, due to the rotation of the particles. So particles that are subject to rotation may stay in suspension due to the Magnus forces and do not contribute to the deposition. From this it is clear that an increasing flow velocity will result in more erosion, finally resulting in hydraulic transport without a bed. A decreasing flow velocity will result in less erosion and an increasing bed thickness, resulting in the danger of plugging the pipeline.

Shells lying convex upwards on the bed in general are more difficult to erode than sand particles, as long as the sand particles are much smaller than the thickness of the shells. The shells used in the research had a thickness varying from 1.13 to 2.69 mm. So the shells armor the bed and require a higher flow velocity than the original sand bed. Now as long as the bed thickness is not increasing, there is no problem, but since hydraulic transport is not a simple stationary process, there will be moments where the flow may decrease and moments where the density may increase, resulting in an increase of the bed thickness. Since the shells are armoring the bed, there will not be a decrease of the bed thickness at moments where the flow is higher or the density is lower, which would be the case if the bed consists of just sand particles. So there is a danger of a bed thickness increasing all the time and finally plugging the pipeline. The question arises, how much we have to increase the flow or flow velocity in order to erode the top layer of the bed where the shells are armoring the bed.

From the research of Dey (2003) it appears that the bottom shear stress to erode the shells varies from 0.56-0.72 Pa for a bed with 0.8 mm sand and from 0.37-0.61 Pa for a bed with 0.3 mm sand. It should be noted that these are shear stresses averaged over a large number of observations and that individual experiments have led to smaller and bigger shear stresses. So the average shear stresses decrease slightly with a decreasing sand particle size due to the change in velocity distribution. These shear stresses require average flow velocities that are 11%-26% higher than the flow velocities required to erode the 0.8 mm sand bed and 48%-89% higher to erode the 0.3 mm sand bed.
From these numbers it can be expected that the shear stresses required to erode the shells, match the shear stresses required to erode a bed with sand grains of 1-1.5 mm and it is thus advised to apply the LDV of 1-1.5 mm sand grains in the case of dredging a sand containing a high percentage of shells, in the case the shells are not too much fragmented.

### 5.2.12 Nomenclature Hydraulic Transport of Sand/Shell Mixtures

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_0$</td>
<td>Pivot angle at $d^+=1$</td>
<td>°</td>
</tr>
<tr>
<td>$C_D$</td>
<td>Drag coefficient</td>
<td>-</td>
</tr>
<tr>
<td>$d$</td>
<td>Diameter of particle or sphere</td>
<td>m</td>
</tr>
<tr>
<td>$d^+$</td>
<td>Dimensionless particle diameter</td>
<td>-</td>
</tr>
<tr>
<td>$E$</td>
<td>Exposure level</td>
<td>-</td>
</tr>
<tr>
<td>$k_S$</td>
<td>Bed roughness</td>
<td>m</td>
</tr>
<tr>
<td>$k^+_S$</td>
<td>Dimensionless bed roughness</td>
<td>m</td>
</tr>
<tr>
<td>LDV</td>
<td>Limit Deposit Velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$P$</td>
<td>Probability related to transition smooth/rough</td>
<td>-</td>
</tr>
<tr>
<td>$Re_p$</td>
<td>Particle Reynolds number</td>
<td>-</td>
</tr>
<tr>
<td>$Re^*$</td>
<td>Boundary Reynolds number</td>
<td>-</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
<td>K</td>
</tr>
<tr>
<td>$u^*$</td>
<td>Friction velocity</td>
<td>m/s</td>
</tr>
</tbody>
</table>
\[ u \quad \text{Velocity} \quad \text{m/s} \]

\[ u_{\text{top}}^+ \quad \text{Dimensionless velocity at top of particle} \quad - \]

\[ u_{\text{eff}}^+ \quad \text{Dimensionless effective turbulent added velocity} \quad - \]

\[ U \quad \text{Average velocity above the bed} \quad \text{m/s} \]

\[ V_{\text{imin}} \quad \text{Minimum friction velocity} \quad \text{m/s} \]

\[ y_{\text{top}} \quad \text{Height of particle} \quad \text{m} \]

\[ y_{\text{top}}^+ \quad \text{Dimensionless height of particle} \quad - \]

\[ z^+ \quad \text{Coefficient} \quad - \]

\[ \alpha \quad \text{Shell shape factor} \quad - \]

\[ \alpha_{\text{Lam}} \quad \text{Laminar velocity function} \quad - \]

\[ \alpha_{\text{Turb}} \quad \text{Turbulent velocity function} \quad - \]

\[ \delta_v \quad \text{Thickness of the viscous sub-layer} \quad \text{m} \]

\[ \delta_v^+ \quad \text{Dimensionless thickness of the viscous sub-layer} \quad - \]

\[ \kappa \quad \text{Von Karman constant} \quad 0.412 \]

\[ \lambda_l \quad \text{Friction coefficient (see Moody diagram)} \quad - \]
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_l$</td>
<td>Liquid density</td>
<td>ton/m$^3$</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Shape factor particle</td>
<td>-</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Pivot angle</td>
<td>°</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Shields parameter</td>
<td>-</td>
</tr>
<tr>
<td>$\theta_{cr}$</td>
<td>Critical Shield parameter, initiation of motion</td>
<td>-</td>
</tr>
<tr>
<td>$\mu_{sf}$</td>
<td>Sliding friction coefficient</td>
<td>-</td>
</tr>
</tbody>
</table>