2.4: Other Important Parameters

2.4.1 The Slip Velocity and the Slip Ratio

The slip velocity \( v_{sl} \) is the difference between the velocity of the solids \( v_s \) and the velocity of the liquid \( v_l \) or the velocity of the mixture, the line speed \( v_{ls} \). For small volumetric concentrations the velocity of the liquid almost equals the line speed, but for higher volumetric concentrations there is a difference. It should also be noted that not all particles have the same velocity. Part of the solids may be in a stationary or sliding bed, while another part is suspended in the liquid above the bed. In this book the slip velocity is related to the line speed.

\[
v_{sl} = v_{ls} - v_s = v_{ls} \cdot \left(1 - \frac{v_s}{v_{ls}}\right)
\]

The slip ratio \( \xi \) is the ratio of the slip velocity \( v_{sl} \) to the line speed \( v_{ls} \). A slip ratio of 0 means that the particles have the same velocity as the liquid. A slip ratio of 1 means that the particles have a velocity of zero.

\[
\xi = \frac{v_{sl}}{v_{ls}}
\]

2.4.2 The Spatial and Delivered Volumetric Concentration

The spatial volumetric concentration \( C_{vs} \) is the volume occupied by the solids \( V_s \) divided by the total mixture volume of a pipe segment \( V_m \).

\[
C_{vs} = \frac{v_s}{V_m}
\]
The delivered or transport volumetric concentration \( C_{vt} \) is the volume flow of solids \( Q_s \) divided by the total mixture volume flow \( Q_m \).

\[
C_{vt} = \frac{Q_s}{Q_m}
\]

For a certain control volume the volumetric transport concentration \( C_{vt} \) can be determined if the volumetric spatial concentration \( C_{vs} \) and the slip velocity \( v_{sl} \) are known, given a certain line speed \( v_{ls} \).

\[
C_{vt} \left(1 - \frac{v_{sl}}{v_{ls}}\right) = (1 - \xi) \cdot C_{vs}
\]

Likewise, for a certain control volume, the volumetric spatial concentration \( C_{vs} \) can be determined if the volumetric transport concentration \( C_{vt} \) and the slip velocity \( v_{sl} \) are known, given a certain line speed \( v_{ls} \).

\[
C_{vs} = \frac{v_{ls}}{v_{ls} - v_{sl}} \cdot C_{vt} = \left(\frac{1}{1 - \xi}\right) \cdot C_{vt}
\]

A special spatial volumetric concentration is the bed concentration. In soil mechanics the porosity is mostly used, in slurry transport the concentration. Porosities may vary from 40% for very dense sand to 50%. For very loose sand, resulting in bed concentrations \( C_{vb} \) of 50% for very loose sand to 60% for very dense sand.

### 2.4.3 Densities

When transporting solids mixed with a liquid, the mixture density \( \rho_m \) can be determined from the solids density \( \rho_s \) and liquid density \( \rho_l \) if the spatial volumetric concentration \( C_{vs} \) of the solids is known, according to:

\[
\rho_m = C_{vs} \cdot \rho_s + \left(1 - C_{vs}\right) \cdot \rho_l
\]

If the mixture density \( \rho_m \) is known, the spatial volumetric concentration \( C_{vs} \) can be determined from the mixture density \( \rho_m \), the solids density \( \rho_s \) and the liquid density \( \rho_l \) according to:

\[
C_{vs} = \frac{\rho_m - \rho_l}{\rho_s - \rho_l}
\]

The specific gravity is the ratio of a solids \( \rho_s \) or mixture \( \rho_m \) density to the water density, giving:

\[
SG = \frac{\rho_s}{\rho_l} \quad \text{or} \quad \frac{\rho_m}{\rho_l} \quad \text{often referred to as: } \quad S_s = \frac{\rho_s}{\rho_l} \quad \text{and } \quad S_m = \frac{\rho_m}{\rho_l}
\]

The two main densities considered are the density of water and the density of quarts. The density of water is about 1000 kg/m\(^3\) or 1 ton/m\(^3\). The density of water depends on the salinity and the temperature and may vary between 958 kg/m\(^3\) (sweet water at 100 degrees centigrade) and 1030 kg/m\(^3\) (salt water at about 10 degrees centigrade). In this book often
a value of 1025 kg/m³ is used for salt water. The density of quarts (sand and gravel) is about 2650 kg/m³ or 2.65 ton/m³.

### 2.4.4 The Relative Submerged Density $R_{sd}$

The relative submerged density $R_{sd}$ is defined as:

$$R_{sd} = \frac{\rho_s - \rho_l}{\rho_l}$$

### 2.4.5 Viscosities

The dynamic (shear) viscosity of a fluid expresses its resistance to shearing flows, where adjacent layers move parallel to each other with different speeds. It can be defined through the idealized situation known as a Couette flow, where a layer of fluid is trapped between two horizontal plates, one fixed and one moving horizontally at constant speed $u$. (The plates are assumed to be very large, so that one need not consider what happens near their edges.)

If the speed of the top plate is small enough, the fluid particles will move parallel to it, and their speed will vary linearly from zero at the bottom to $u$ at the top. Each layer of fluid will move faster than the one just below it, and friction between them will give rise to a force resisting their relative motion. In particular, the fluid will apply on the top plate a force in the direction opposite to its motion, and an equal but opposite one to the bottom plate. An external force is therefore required in order to keep the top plate moving at constant speed.

The magnitude $F$ of this force is found to be proportional to the speed $u$ and the area $A$ of each plate, and inversely proportional to their separation $y$:

$$F = \frac{\mu_l \cdot A \cdot u}{y}$$

The proportionality factor $\mu_l$ in this formula is the viscosity (specifically, the dynamic viscosity) of the fluid. The ratio $u/y$ is called the rate of shear deformation or shear velocity, and is the derivative of the fluid speed in the direction perpendicular to the plates. Isaac Newton expressed the viscous forces by the differential equation

$$\tau = \frac{\mu_l}{\rho_l} \frac{\partial u}{\partial y}$$

Where $\tau = F/A$ and $\frac{\partial u}{\partial y}$ is the local shear velocity. This formula assumes that the flow is moving along parallel lines and the $y$ axis, perpendicular to the flow, points in the direction of maximum shear velocity. This equation can be used where the velocity does not vary linearly with $y$, such as in fluid flowing through a pipe.

Use of the Greek letter mu ($\mu$) for the dynamic stress viscosity is common among mechanical and chemical engineers, as well as physicists. However, the Greek letter eta ($\eta$) is also used by chemists, physicists, and the IUPAC.

The kinematic viscosity (also called "momentum diffusivity") is the ratio of the dynamic viscosity $\mu_l$ to the density of the fluid (here liquid) $\rho_l$. It is usually denoted by the Greek letter nu ($\nu$).

$$\nu = \frac{\mu_l}{\rho_l} \quad \text{or} \quad \mu_l = \nu \cdot \rho_l$$
It is a convenient concept when analyzing the Reynolds number that expresses the ratio of the inertial forces to the viscous forces:

$$\mathrm{Re} = \frac{\rho \cdot \mathbf{u} \cdot L}{\mu} = \frac{\mathbf{u} \cdot L}{v}$$

Where $L$ is a typical length scale in the system.

The dimension of dynamic viscosity in SI units is Poiseuille (Pa·s) and in cgs units Poise (P) or N·s/m² or kg/(m·s).

For example, the dynamic viscosity of water at 20 degrees centigrade is $\mu_l=0.0012$ Pa·s.

The dimension of kinematic viscosity in SI units is m²/s and in cgs units Stokes (St). Typically for water values in the range of $\nu_l=0.000001$ m²/s and $\nu_l=0.0000013$ m²/s are used.

The dynamic viscosity of water can be estimated by, with the temperature in centigrade:

$$\mu_l = \frac{0.10}{2.1482 \cdot ((T-8.435)+\sqrt{8078.4+(T-8.435)^2})-120}$$

The kinematic viscosity of the water is temperature dependent. If a temperature of 10 °C is used as a reference, then the viscosity increases by 27% at 0 °C and it decreases by 30% at 20 °C centigrade. For the kinematic viscosity the following equation is often used:

$$\nu_l = \frac{497 \cdot 10^{-6}}{(42.5+T)^{1.5}}$$

<table>
<thead>
<tr>
<th>Temperature °C</th>
<th>Dynamic Viscosity (Pa·s, N·s/m²) $\times 10^{-3}$</th>
<th>Kinematic Viscosity (m²/s) $\times 10^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.787</td>
<td>1.787</td>
</tr>
<tr>
<td>5</td>
<td>1.519</td>
<td>1.519</td>
</tr>
<tr>
<td>10</td>
<td>1.307</td>
<td>1.307</td>
</tr>
<tr>
<td>20</td>
<td>1.002</td>
<td>1.004</td>
</tr>
</tbody>
</table>

Table 2.4-1: Dynamic and kinematic viscosity of water.
Soils consist of a mixture of particles of different size, shape and mineralogy. Because the size of the particles obviously has a significant effect on the soil behavior, the grain size and grain size distribution are used to classify soils. The grain size distribution describes the relative proportions of particles of various sizes. The grain size is often visualized in a cumulative distribution graph which, for example, plots the percentage of particles finer than a given size as a function of size. The median grain size, $d_{50}$, is the size for which 50% of the particle mass consists of finer particles. Soil behavior, especially the hydraulic conductivity, tends to be dominated by the smaller particles; hence, the term "effective size", denoted by $d_{10}$, is defined as the size for which 10% of the particle mass consists of finer particles.

Sands and gravels that possess a wide range of particle sizes with a smooth distribution of particle sizes are called well graded soils. If the soil particles in a sample are predominantly in a relatively narrow range of sizes, the soil is called uniformly graded soil. If there are distinct gaps in the gradation curve, e.g., a mixture of gravel and fine sand, with no coarse sand, the soils may be called gap graded. Uniformly graded and gap graded soils are both considered to be poorly graded. There are many methods for measuring particle size distribution. The two traditional methods used in geotechnical engineering are sieve analysis and hydrometer analysis.

The size distribution of gravel and sand particles are typically measured using sieve analysis. The formal procedure is described in ASTM D6913-04(2009). A stack of sieves with accurately dimensioned holes between a mesh of wires is used to separate the particles into size bins. A known volume of dried soil, with clods broken down to individual particles,
is put into the top of a stack of sieves arranged from coarse to fine. The stack of sieves is shaken for a standard period of time so that the particles are sorted into size bins. This method works reasonably well for particles in the sand and gravel size range. Fine particles tend to stick to each other, and hence the sieving process is not an effective method. If there are a lot of fines (silt and clay) present in the soil it may be necessary to run water through the sieves to wash the coarse particles and clods through.

A variety of sieve sizes are available. The boundary between sand and silt is arbitrary. According to the Unified Soil Classification System, a #4 sieve (4 openings per inch) having 4.75 mm opening size separates sand from gravel and a #200 sieve with an 0.075 mm opening separates sand from silt and clay. According to the British standard, 0.063 mm is the boundary between sand and silt, and 2 mm is the boundary between sand and gravel.

The classification of fine-grained soils, i.e., soils that are finer than sand, is determined primarily by their Atterberg limits, not by their grain size. If it is important to determine the grain size distribution of fine-grained soils, the hydrometer test may be performed. In the hydrometer tests, the soil particles are mixed with water and shaken to produce a dilute suspension in a glass cylinder, and then the cylinder is left to sit. A hydrometer is used to measure the density of the suspension as a function of time. Clay particles may take several hours to settle past the depth of measurement of the hydrometer. Sand particles may take less than a second. Stoke's law provides the theoretical basis to calculate the relationship between sedimentation velocity and particle size. ASTM provides the detailed procedures for performing the Hydrometer test.

Clay particles can be sufficiently small that they never settle because they are kept in suspension by Brownian motion, in which case they may be classified as colloids.

Table 2.4-2: Soil Classification (combined from different sources)

<table>
<thead>
<tr>
<th>Type of Soil</th>
<th>Particle size (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay</td>
<td>&lt; 0.002</td>
</tr>
<tr>
<td>Fine Silt</td>
<td>0.002–0.006</td>
</tr>
<tr>
<td>Medium Silt</td>
<td>0.006–0.02</td>
</tr>
<tr>
<td>Coarse Silt</td>
<td>0.02–0.06</td>
</tr>
<tr>
<td>Very Fine Sand</td>
<td>0.06–0.10</td>
</tr>
<tr>
<td>Fine Sand</td>
<td>0.10–0.20</td>
</tr>
</tbody>
</table>
Medium Sand 0.20–0.60

Coarse Sand 0.60–1.00

Very Coarse Sand 1.00–2.00

Fine Gravel 2-6

Medium Gravel 6-20

Coarse Gravel 20-60

Cobbles 60-200

Boulders >200

Figure 2.4-1: Some grain distributions of the loading process of a Trailing Suction Hopper Dredge.

Figure 2.4-2: The particle size distributions of the sands used by Roberts et al. (1998).
Table 2.4-2 gives a classification of sands and gravels. Figure 2.4-1 shows the PSD’s of the loading process of a Trailing Suction Hopper Dredger. Three curves are shown, the PSD of the dredged material, the PSD of the material settled in the hopper and the PSD of the material leaving the hopper through the overflow. Figure 2.4-2 shows a number of PSD’s of the research of Roberts et al. (1998), investigating the influence of very fine particles on the initiation of motion (see Miedema (2013)). Table 15.3-1 shows commercial sieve mesh dimensions.

### 2.4.7 The Angle of Internal Friction

Angle of internal friction for a given soil is the angle on the graph (Mohr's Circle) of the shear stress and normal effective stresses at which shear failure occurs. Angle of Internal Friction, $\phi$, can be determined in the laboratory by the Direct Shear Test or the Triaxial Stress Test. Typical relationships for estimating the angle of internal friction, $\phi$, are as follows:

Table 2.4-3: Empirical values for $\phi$, of granular soils based on the standard penetration number, (from Bowels, Foundation Analysis).

<table>
<thead>
<tr>
<th>SPT Penetration, N-Value (blows/foot)</th>
<th>$\phi$ (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>25-30</td>
</tr>
<tr>
<td>4</td>
<td>27-32</td>
</tr>
<tr>
<td>10</td>
<td>30-35</td>
</tr>
<tr>
<td>30</td>
<td>35-40</td>
</tr>
<tr>
<td>50</td>
<td>38-43</td>
</tr>
</tbody>
</table>
Table 2.4-4: Relationship between $\phi$, and standard penetration number for sands, (from Peck 1974, Foundation Engineering Handbook).

<table>
<thead>
<tr>
<th>SPT Penetration, N-Value (blows/ foot)</th>
<th>Density of Sand</th>
<th>$\phi$ (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;4</td>
<td>Very loose</td>
<td>&lt;29</td>
</tr>
<tr>
<td>4 - 10</td>
<td>Loose</td>
<td>29 - 30</td>
</tr>
<tr>
<td>10 - 30</td>
<td>Medium</td>
<td>30 - 36</td>
</tr>
<tr>
<td>30 - 50</td>
<td>Dense</td>
<td>36 - 41</td>
</tr>
<tr>
<td>&gt;50</td>
<td>Very dense</td>
<td>&gt;41</td>
</tr>
</tbody>
</table>

Table 2.4-5: Relationship between $\phi$, and standard penetration number for sands, (from Meyerhof 1956, Foundation Engineering Handbook).

<table>
<thead>
<tr>
<th>SPT Penetration, N-Value (blows/ foot)</th>
<th>Density of Sand</th>
<th>$\phi$ (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;4</td>
<td>Very loose</td>
<td>&lt;30</td>
</tr>
<tr>
<td>4 - 10</td>
<td>Loose</td>
<td>30 - 35</td>
</tr>
<tr>
<td>10 - 30</td>
<td>Medium</td>
<td>35 - 40</td>
</tr>
<tr>
<td>30 - 50</td>
<td>Dense</td>
<td>40 - 45</td>
</tr>
<tr>
<td>&gt;50</td>
<td>Very dense</td>
<td>&gt;45</td>
</tr>
</tbody>
</table>

The angle of internal friction is very important when considering a stationary or sliding bed and sheet flow. Since the bed in a pipeline is formed by sedimentation and usually does not have a long history, the bed density or concentration will be relatively low, resulting in very loose to loose sands or gravels. An angle of internal friction of 30o-35o may be expected, resulting in an internal friction coefficient of 0.577-0.700. Of course there may always be exceptions.

Figure 2.4-3 shows the angle of repose of granular materials, which basically is the smallest internal friction angle of a
granular material. It is clear from this figure that this angle increases with the particle diameter.

Figure 2.4-3: Angle of repose for granular materials (Simons, 1957).

```
![](https://eng.libretexts.org/Bookshelves/Civil_Engineering/Book%3A_Slurry_Transport_(Miedema)/02%3A_Dimensionless_Num...)
```

2.4.8 The Angle of External Friction

The external friction angle, \( \delta \), or friction between a soil medium and a material such as the composition from a retaining wall or pile may be expressed in degrees as the following:

Table 2.4-6: External friction angle \( \varphi \) values.

<table>
<thead>
<tr>
<th>Material</th>
<th>External Friction Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>20° steel piles (NAVFAC)</td>
<td>20°</td>
</tr>
<tr>
<td>0.67·( \varphi ) to 0.83·( \varphi ) USACE</td>
<td>0.67·( \varphi ) to 0.83·( \varphi )</td>
</tr>
<tr>
<td>20° steel (Broms)</td>
<td>20°</td>
</tr>
<tr>
<td>3/4·( \varphi ) concrete (Broms)</td>
<td>3/4·( \varphi )</td>
</tr>
<tr>
<td>2/3·( \varphi ) timber (Broms)</td>
<td>2/3·( \varphi )</td>
</tr>
<tr>
<td>0.67·( \varphi ) Lindeburg</td>
<td>0.67·( \varphi )</td>
</tr>
<tr>
<td>2/3·( \varphi ) for concrete walls (Coulomb)</td>
<td>2/3·( \varphi )</td>
</tr>
</tbody>
</table>

The external friction angle can be estimated as \( 1/3·\varphi \) for smooth retaining walls like sheet piles or concrete surfaces against timber formwork, or as \( 1/2·\varphi \) to \( 2/3·\varphi \) for rough surfaces. In the absence of detailed information the assumption of \( 2/3·\varphi \) is commonly made.
The angle of external friction is very important when considering a stationary or sliding bed. Since the bed in a pipeline is formed by sedimentation and usually does not have a long history, the bed density or concentration will be relatively low, resulting in very loose to loose sands or gravels. An angle of external friction of 20°-24° may be expected, resulting in an external friction coefficient of 0.364-0.445. Of course there may always be exceptions.

Based on Figure 2.4-3 one may expect an increasing external friction angle/coefficient with increasing particle diameter.