10.1: Head Loss Equation

In dredging and other industries it is important to be able to predict the required pressure and power to transport solid-liquid mixtures over a short to very long distance, with or without pipe inclination and/or elevation in a system with one or more pumps (usually centrifugal pumps). The pipeline resistance, pressure difference between entrance (suction mouth) and discharge, is an ascending curve, ascending with increasing flow. The pump pressure curve is usually a second degree descending curve, descending with increasing flow. The two curves will intersect at a certain flow rate, meaning that the pressure required to transport the mixture through the pipeline equals the available pump pressure. This is called the working point. At lower flows the available pump pressure is higher than the pressure required to transport the mixture through the pipeline, meaning that the mixture in the pipe will accelerate until the working point is reached. At higher flows the available pump pressure is lower than the pressure required to transport the mixture through the pipeline, meaning that the mixture in the pipe will decelerate until the working point is reached. The second degree pump curve, second degree with the flow rate $Q_m$, also depends on the mixture density $\rho_m$, usually a linear relation, and depends on the particle diameter $d$. The latter is because large particles influence the efficiency of the pump. Figure 10.1-1 shows a pump/pipeline system with multiple centrifugal pumps. The question is now, how can the theory developed in this book, the DHLLDV Framework, be applied to such a pump/pipeline system?

**Figure 10.1-1: A pump pipeline system with boosters.**
To answer this question, first the total pipeline resistance is analyzed. The pressure losses from the suction mouth (the entrance) to the end of the pipeline (the discharge), assuming many fittings in the pipeline, are, assuming one pipe diameter for all \( n \) pipe segments, one mixture density in the whole pipeline and a stationary process (not time or position dependent):

\[
\Delta p_m = \frac{1}{2} \cdot \rho_m \cdot v_{ls}^2 + \lambda_l \cdot \frac{L_{tot}}{D_p} \cdot \frac{1}{2} \cdot \rho_l \cdot v_{ls}^2 + \rho_l \cdot g \cdot L_{tot} \cdot R_{sd} \cdot C_{vt} \cdot E_{rhg} + \frac{1}{2} \cdot \rho_m \cdot v_{ls}^2 \cdot \sum_{i=1}^{n} \left( \sum_{l=1}^{m_i} \left( \xi_{l,i} \right) \right) + \rho_m \cdot g \cdot \left( H_{n,o} - H_{0,i} \right)
\]

In this example it is assumed that the suction pipe and the discharge pipe have the same diameter. In reality this is often not the case. The suction pipe is usually a bit larger than the discharge pipe. However for this example it does not make a lot of difference.

The pressure loss equation consists of 5 terms:

1. The 1st term represents the acceleration losses. Outside the pipe the mixture is assumed not to have a velocity, but inside the pipe it has the line speed. So the mixture has to be accelerated from zero velocity to the line speed, resulting in some pressure loss. Basically this can be determined from the Bernoulli equation. For the total pressure loss this is negligible, but for the calculation of possible cavitation at the inlet of the first pump it is.

2. The 2nd term is the so called Darcy-Weisbach term for straight pipeline resistance of pure liquid (water in this case). The Darcy-Weisbach friction factor can be determined from the Moody diagram. For large diameter pipes and high line speeds the Darcy Weisbach friction factor is close to \( \lambda_l = 0.01 \).

3. The 3rd term is the solids effect based on the DHLLDV Framework.

4. The 4th term represents the pressure losses due to many fittings, telescopes and so on.

5. The 5th term represents the total elevation pressure loss, from suction mouth to discharge.
Figure 10.1-2 shows two pump curves, one for pure liquid (water, the blue descending line) and one for a mixture density $\rho_m$ of about 1.4 tons/m$^3$ (the brown descending line). The pipeline resistance curve for pure liquid (water, the blue ascending line) and the ELM curve (the brown ascending line). For the ELM curve it is assumed that the water density can be replaced by the mixture density and there is no further solids effect. The thick red line gives the result of equation (10.1-1) using the DHLLDV Framework for heterogeneous and homogeneous transport. The two dashed lines give the result of equation (10.1-1) using the Jufin-Lopatin (1966) model and the Wilson et al. (1992) heterogeneous model. The 4 colored circles give the 4 possible working points.

1. Yellow, the pipeline is 100% full of water and the pump is full of water.
2. Red, the pipeline is close to 100% full of water, but the pump is full of mixture.
3. Blue, the pipeline is 100% full of mixture and the pump is full of mixture.
4. Brown, the pipeline is close to 100% full of mixture, but the pump is full of water.

The Jufin-Lopatin (1966) model and the Wilson et al. (1992) heterogeneous model are also shown here to illustrate that different models may give different working (intersection) points. In real life there is not just one working point, but there is a working area, formed by the 4 points in Figure 10.1-2 under the assumption that the graph is made for the maximum achievable mixture density.

The mixture resistance of the DHLLDV Framework (the thick red line) has a vertical distance compared with the water resistance curve. This distance depends on the mixture flow. This distance is the result of the so called solids effect which is the third term in equation (10.1-1). This solids effect is modeled in this book. On very short pipelines this solids effect is not to important. The first, fourth and fifth terms in equation (10.1-1) dominate the pressure losses. The second and third terms contain the length of the pipeline and will thus be small compared to the other terms. In long pipelines however, the second and third terms will dominate. The influence of the pipe elevation completely depends on the value of the pipe elevation.

Figure 10.1-3: The hydraulic gradient for a 0.762 m (30 inch) pipe and 9 particle diameters for a constant delivered volumetric concentration of 17.5%.
Figure 10.1-4: The relative excess hydraulic gradient for a 0.762 m (30 inch) pipe and 9 particle diameters for a constant delivered volumetric concentration of 17.5%.

Often the pressure losses are expressed in terms of the hydraulic gradient. The hydraulic gradient is the pressure loss divided by the density of the liquid (water, 1-1.025 ton/m$^3$), the gravitational constant (9.81 m/s$^2$) and the length of the pipeline. Figure 10.1-3 shows the hydraulic gradient in a $D_p=0.762$ m (30 inch) pipe and a delivered volumetric concentration of 17.5% for 9 particle diameters. It is clear from this figure that the hydraulic gradient depends strongly on the particle diameter.

Figure 10.1-4 shows the relative excess hydraulic gradient $E_{rhg}$ for different particle diameters in a $D_p=0.762$ m (30 inch) pipe as a function of the hydraulic gradient $i_l$. The graph is also constructed for a 17.5% constant delivered volumetric concentration. For other concentrations the graph can also be used for the heterogeneous (downwards) and the homogeneous (upwards) regimes (so if the slip can be neglected), but not for the sliding bed regime (almost horizontal left top).