4.5: Gradient

The gradient operator is an important and useful tool in electromagnetic theory. Here’s the main idea:

The \textit{gradient} of a scalar field is a vector that points in the direction in which the field is most rapidly increasing, with the scalar part equal to the rate of change.

A particularly important application of the gradient is that it relates the electric field intensity \(\mathbf{E}(\mathbf{r})\) to the electric potential field \(V(\mathbf{r})\). This is apparent from a review of Section 2.2; see in particular, the battery-charged capacitor example. In that example, it is demonstrated that:

- The \textit{direction} of \(\nabla V(\mathbf{r})\) is the direction in which \(V(\mathbf{r})\) decreases most quickly, and
- The \textit{scalar part} of \(\nabla V(\mathbf{r})\) is the rate of change of \(V(\mathbf{r})\) in that direction. Note that this is also implied by the units, since \(V(\mathbf{r})\) has units of V whereas \(\nabla V(\mathbf{r})\) has units of V/m.

The gradient is the mathematical operation that relates the vector field \(\mathbf{E}(\mathbf{r})\) to the scalar field \(V(\mathbf{r})\) and is indicated by the symbol \(\nabla\) as follows: \(\nabla V(\mathbf{r}) = \nabla V(\mathbf{r})\) or, with the understanding that we are interested in the gradient as a function of position \(\mathbf{r}\), simply \(\nabla V = \nabla V\).

At this point we should note that the gradient is a very general concept, and that we have merely identified one application of the gradient above. In electromagnetics there are many situations in which we seek the gradient \(\nabla f(\mathbf{r})\) of some scalar field \(f(\mathbf{r})\). Furthermore, we find that other differential operators that are important in electromagnetics can be interpreted in terms of the gradient operator \(\nabla\). These include \textit{divergence} (Section 4.6), \textit{curl} (Section 4.8), and the \textit{Laplacian} (Section 4.10).

In the Cartesian system:

\[
\nabla f = \hat{x} \frac{\partial f}{\partial x} + \hat{y} \frac{\partial f}{\partial y} + \hat{z} \frac{\partial f}{\partial z}
\]
Example $\PageIndex{1}$: Gradient of a ramp function.

Find the gradient of $f=ax$ (a "ramp" having slope $a$ along the $x$ direction).

Solution

Here, $\frac{\partial f}{\partial x} = a$ and $\frac{\partial f}{\partial y} = \frac{\partial f}{\partial z} = 0$. Therefore $\nabla f = \hat{x}a$. Note that $\nabla f$ points in the direction in which $f$ most rapidly increases, and has magnitude equal to the slope of $f$ in that direction.

The gradient operator in the cylindrical and spherical systems is given in Appendix B2.

Contributors and Attributions