3: Ideal Diode Equation

As seen in the previous sections, a p-n junction diode creates the following current: under reverse bias, there is a small, constant reverse current, and under forward bias, there is a forward current that increases with voltage. The current-voltage function (also called the “i-\(v\) characteristic”) for an ideal diode is

\[i(v) = I_S \left[\exp \left(\frac{v}{\eta V_T}\right) - 1\right], \quad v > V_Z \tag{eq1}\]

- where \(I_S\) is the reverse saturation current,
- \(v\) is the applied voltage (reverse bias is negative),
- \(V_T = T / 11,586\) is the volt equivalent of temperature, and
- \(\eta\) is the emission coefficient, which is 1 for germanium devices and 2 for silicon devices.

Note that \(i\) is defined as positive when flowing from \(p\) to \(n\). Equation \(\text{ref(eq1)}\) is also called the Shockley ideal diode equation or the diode law. Note also that for \(v \leq V_Z\), the diode is in breakdown and the ideal diode equation no longer applies; for \(v \leq V_Z\), \(\text{quad } i = -\infty\). The ideal diode i-\(v\) characteristic curve is shown below:
The ideal diode equation is very useful as a formula for current as a function of voltage. However, at times the inverse relation may be more useful; if the ideal diode equation is inverted and solved for voltage as a function of current, we find:

\[ v(i) = \eta V_T \ln \left[ \left( \frac{i}{I_S} \right) + 1 \right]. \]

### Approximations

#### Infinite step function

A number of approximations of diode behavior can be made from the ideal diode equation. The simplest approximation to make is to represent the diode as a device that allows no current through -- that is, it acts as an open circuit -- under reverse bias, and allows an unlimited amount of current through -- a closed circuit -- under forward bias. In this simplified model, the current-voltage relation (also called the "i-v characteristic") is an infinite step function:

\[ i = \begin{array}{l}
0, \; v \leq 0 \\
\infty, \; v > 0
\end{array} \]

This characteristic is depicted below:
This approximation is used in circuit analysis, as we will see in the next section.

**Forward current approximation**

In the case of large forward bias, a good approximation of the ideal diode equation is to simply set the second term of Equation \ref{eq1} to zero. This approximation is valid because the ideal diode i-v curve increases very quickly, and because reverse saturation current IS is typically very small. This approximation is acceptable for \( v > 0.2 \, V \). The forward current approximation, as we will call it, results in the following formula:

\[
\begin{align*}
  i(v) & \approx I_S \exp \left( \frac{v}{\eta V_T} \right) \\
  & \quad \text{for } v > 0.2 \, V.
\end{align*}
\]

**Reverse current approximation**

Under reverse bias, the resulting current can be treated as simply the reverse saturation current, \( I_S \). In reality, the current under reverse bias will asymptotically approach \( I_S \), but the small magnitude of the reverse saturation current makes this discrepancy negligible. The reverse current approximation is valid over the range \( V_Z < v < 0 \) (the diode enters breakdown for \( v \leq V_Z \)):

\[
\begin{align*}
  i(v) & \approx I_S \\
  & \quad \text{for } V_Z < v < 0.
\end{align*}
\]

**References**