6.1.2: External Forces

First, the terms on the left hand side, or the forces, have to be discussed. The forces, excluding the external forces, are the body forces, and the surface forces as the following

\[
\mathbf{F}_{\text{total}} = \mathbf{F}_b + \mathbf{F}_s
\]
\[\text{(1)}\]

\[
\sum \mathbf{F}_b = \int_{\text{sys}} \mathbf{g} \rho \, dV
\]
\[\text{(2)}\]

which acts through the mass center towards the center of earth. After infinitesimal time the gravity force acting on the system is the same for control volume, hence,

\[
\int_{\text{sys}} \mathbf{g} \rho \, dV = \int_{\text{cv}} \mathbf{g} \rho \, dV
\]
\[\text{(3)}\]

The integral yields a force through the center mass which has to be found separately.
In this chapter, the surface forces are divided into two categories: one perpendicular to the surface and one with the surface direction (in the surface plain see Figure 6.1.). Thus, it can be written as

\[
\label{mom:eq:cFs}
\sum \pmb{F}_s = \int _{c.v.} \pmb{S_n}\,dA + \int _{c.v.} \pmb{\tau}\,dA
\]

\[
\label{mom:eq:pSn}
\pmb{S}_n = -\pmb{P}\,\hat{n} + \overbrace{\pmb{S_{\nu}}}^{\sim 0}
\]

Where \(\pmb{S_{\nu}}\) is perpendicular stress due to viscosity. Again, \(\hat{n}\) is an unit vector outward of element area and the negative sign is applied so that the resulting force acts on the body.

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