9.1.2: Theory Behind Dimensional Analysis

In chemistry it was recognized that there are fundamental elements that all the material is made from (the atoms). That is, all the molecules are made from a combination of different atoms. Similarly to this concept, it was recognized that in many physical systems there are basic fundamental units which can describe all the other dimensions or units in the system. For example, isothermal single component systems (which does not undergo phase change, temperature change and observed no magnetic or electrical effect) can be described by just The units or dimensions are, time, length, mass, quantity of substance (mole). For example, the dimension or the units of force can be constructed utilizing Newton's second law i.e.~mass times acceleration $\longrightarrow m\,a = M\,L/t^2$. Increase of degree of freedom, allowing this system to be non–isothermal will increase only by one additional dimension of temperature, θ. These five fundamental units are commonly the building blocks for most of the discussion in fluid mechanics (see Table of basic units 9.1)

Basic Units of Common System					
Standard System			**Old System**		
Name	**Latter**	**Units**	**Name**	**Latter**	**Units**
Mass	M	kg	Force	F	N
Length	L	m	Length	L	m
Time	t	sec	Time	t	sec
Temperature	θ	$\,^{\circ}C$	Temperature	T	$\,^{\circ}C$

Additional Basic Units for Magnetohydrodynamics					
Electric Current	\(A\)	\([A]\)mpere	Electric Current	\(A\)	\([A]\)mpere
Luminous Intensity	\(cd\)	\(cd\) candle	Luminous Intensity	\(cd\)	\(cd\) candle
Chemical Reactions					
Quantity of Substance	\(\mathfrak{M}\)	\(mol\)	Quantity of Substance	\(\mathfrak{M}\)	\(mol\)

The choice of these basic units is not unique and several books and researchers suggest a different choice of fundamental units. One common selection is substituting the mass with the force in the previous selection (F, t, L, mol, Temperature). This author is not aware of any discussion on the benefits of one method over the other method. Yet, there are situations in which first method is better than the second one while in other situations, it can be the reverse. In this book, these two selections are presented. Other selections are possible but not common and, at the moment, will not be discussed here.

Example 9.1

What are the units of force when the basic units are: mass, length, time, temperature (M, L, t, \(\theta\))? What are the units of mass when the basic units are: force, length, time, temperature (F, L, t, T)? Notice the different notation for the temperature in the two systems of basic units. This notation has no significance but for historical reasons remained in use.

Solution 9.1

These two systems are related as the questions are the reversed of each other. The connection between the mass and force can be obtained from the simplified Newton's second law \(F = m\, a\) where \(F\) is the force, \(m\) is the mass, and \(a\) is the acceleration. Thus, the units of force are

$$\label{basicMass:force}
F = \dfrac{ M\, L}{ t^2}$$

For the second method the unit of mass are obtain from Equation (1) as

$$\label{basicMass:mass}
M = \dfrac{ F\,t^2}{L}$$

The number of fundamental or basic dimensions determines the number of the combinations which affect the physical situations. The dimensions or units which affect the problem at hand can be reduced because these dimensions are repeating or reoccurring. The Buckingham method is based on the fact that all equations must be consistent with their units. That is the left hand side and the right hand side have to have the same units. Because they have the same units

the equations can be divided to create unitless equations. This idea alludes to the fact that these unitless parameters can be found without any knowledge of the governing equations. Thus, the arrangement of the effecting parameters in unitless groups yields the affecting parameters. These unitless parameters are the dimensional parameters. The following trivial example demonstrates the consistency of units

Example 9.2

Newton's equation has two terms that related to force \(F= m\,a +\dot{m}\, U/). Where \(F\) is force, \(m\) is the mass, \(a\) is the acceleration and dot above \(\dot{m}\) indicating the mass derivative with respect to time. In particular case, this equation get a form of

\[\label{unitsInEq:gov}
F = m \, a + 7
\]
where \(7\) represent the second term. What are the requirement on equation (3)?

Solution 9.2

Clearly, the units of [\(F\)], \(m\,a\) and \(7\) have to be same. The units of force are [\(N\)] which is defined by first term of the right hand side. The same units force has to be applied to \(7\) thus it must be in [\(N\)].

Suppose that there is a relationship between a quantity a under the question and several others parameters which either determined from experiments or theoretical consideration which is of the form

\[\label{dim:eq:generalRelationship}
D = f (a_1, a_2, \cdots, a_i, \cdots, a_n)
\]
where \(D\) is dependent parameters and \(a_1, a_2, \cdots, a_i, \cdots, a_n\) are have independent dimensions. From these independent parameters \(a_1, a_2, \cdots, a_i\) have independent dimensions (have basic dimensions). This mean that all the dimensions of the parameters \(a_{i+1}, \cdots, a_n\) can be written as combination of the independent parameters \(a_1, a_2, \cdots, a_i\). In that case it is possible to write that every parameter in the later set can written as dimensionless

\[\label{dim:eq:laterDimless}
\dfrac{a_{i+1}}
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= \text{dimensionless }
\] The "non-basic" parameter would be dimensionless when divided by appropriately and selectively chosen set of constants \({p_1}, {p_2}, \cdots, p_i\).

Example 9.3

In a experiment, the clamping force is measured. It was found that the clamping force depends on the length of

experimental setup, velocity of the upper part, mass of the part, height of the experimental setup, and leverage the force is applied. Chose the basic units and dependent parameters. Show that one of the dependent parameters can be normalized.

Solution 9.3

The example suggest that the following relationship can be written.

$$\label{dim:eq:appropriatelySelectively}$$
$$F = f (L, U, H, \tau, m)$$
$$\]$$

The basic units in this case are in this case or length, mass, and time. No other basic unit is need to represent the problem. Either L, H, or τ can represent the length. The mass will be represented by mass while the velocity has to be represented by the velocity (or some combination of the velocity). Hence a one possible choice for the basic dimension is L, m, and U. Any of the other Lengths can be reprsented by simple division by the L. For example

$$\label{dim:eq:appropriatelySelectivelyRatio}$$
$$\text{Normalize parameter} = \dfrac{H}{L}$$
$$\] \text{Or the force also can be normalized as}$$

$$\label{dim:eq:appropriatelySelectivelyR2}$$
$$\text{Another Normalize parameter} = \dfrac{F}{m\,U^2\,L^{-1} }$$

\] The acceleration can be any part of acceleration component such as centrifugal acceleration. Hence, the force is mass times the acceleration.

The relationship (4) can be written in the light of the above explanation as

$$\label{dim:eq:gRD}$$
$$\dfrac{D}{ {a_1}^{p_1}, {a_2}^{p_2}, \cdots, {a_i}^{p_1}} = \$$
$$F \left(\dfrac{a_{i+1}}$$

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{{a_{n}}^{p_{n,1}},\,{a_n}^{p_{n, 2} } ,\cdots, \,{a_n}^{p_{n, i} } }
\right)
\]

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