9.2.3.1: One Shot Method: Constructing Dimensionless Parameters

In this method, the solution is obtained by assigning the powers to the affecting variables. The results are used to compare the powers on both sides of the equation. Several examples are presented to demonstrate this method.

Example 9.4

Fig. 9.3 Resistance of infinite cylinder.

The researcher intuition suggests that the resistance to flow, \(R\) is a function of the radius \(r\), the velocity \(U\), the density, \(\rho\), and the absolute viscosity \(\mu\). Based on this limited information construct a relationship of the variables, that is

\[
R = f (r, U, \rho, \mu)
\]
Solution 9.4

The functionality should be in a form of

\[ R = f( r^a, \frac{L}{t}, \rho^c, \mu^d ) \]

The units of the parameters are provided in Table 9.3. Thus substituting the data from the table into equation (10) results in

\[ R = \text{Constant} \left( r \right)^a \left( \frac{L}{t} \right)^b \left( \frac{M}{L^3} \right)^c \left( \frac{M}{L\cdot t} \right)^d \]

From equation (11) the following requirements can be obtained

\[
\begin{array}{lrcl}
time, t & -2 &=& -b - d \\
mass, M & 1 &=& c + d \\
length, L & 1&=& a + b - 3c - d \\
\end{array}
\]

In equations (11) there are three equations and 4 unknowns. Expressing all the three variables in term of \(d\) to obtain

\[
\begin{array}{rcl}
a & = & 2-d \\
b & = & 2 - d \\
c & = & 1 - d \\
\end{array}
\]

Substituting equation (13) into equation results in

\[
\frac{R}{\rho U^2 r^2} = \text{Constant} \left( \frac{\mu}{\rho U r} \right)^d
\]

The relationship between the two sides in equation (15) is related to the two dimensionless parameters. In dimensional analysis the functionality is not clearly defined by but rather the function of the parameters. Hence, a simple way, equation (15) can be represented as
An example of a ship is a typical example where more than one dimensionless is to be constructed. Also introduction of dimensional matrix is presented.

Example 9.5

The modern ship today is equipped with a propeller as the main propulsion mechanism. The thrust, \( T \) is known to be a function of the radius, \( r \), the fluid density, \( \rho \), relative velocity of the ship to the water, \( U \), rotation speed, \( \text{rpm} \) or \( N \), and fluid viscosity, \( \mu \). Assume that no other parameter affects the thrust, find the functionality of these parameters and the thrust.

Solution 9.5

The general solution under these assumptions leads to solution of

\[
T = C \, r^a \, \rho^b \, U^c \, N^d \, \mu^e
\]

It is convenient to arrange the dimensions and basic units in table.

| \(|T|\) | \(|r|\) | \(|\rho|\) | \(|U|\) | \(|N|\) | \(|\mu|\) |
|---|---|---|---|---|---|
| \(|M|\) | 1 | 0 | 1 | 0 | 0 | 1 |
| \(|L|\) | 1 | 1 | -3 | 1 | 0 | -1 |
| \(|t|\) | -2 | 0 | 0 | -1 | -1 | -1 |

Using the matrix results in

\[
M \, L \, t^{-2} = L^a \, \rho^b \, U^c \, N^d \, \mu^e
\]

This matrix leads to three equations.

\[
\text{Mass}, M = c + e \\
\text{Length}, L = a + b - 3c - e \\
\text{time}, t = c - d - e
\]

The solution of this system is
\[a = 2 + d - e\]
\[b = 2 - d - e\]
\[c = 1 - e\]

Substituting the solution (20) into equation yields

\[T = C \cdot r^{(2+d-e)} \cdot \rho^{(2-d-e)} \cdot U^{(1-e)} \cdot N^d \cdot \mu^f\]

Rearranging equation (21) provides

\[T = C \cdot \rho \cdot U^2 \cdot r^2 \left( \frac{\rho \cdot U \cdot r}{\mu} \right)^d \left( \frac{r \cdot N}{U} \right)^e\]

From dimensional analysis point of view the units under the power \((d)\) and \((e)\) are dimensionless. Hence, in general it can be written that

\[\frac{T}{\rho \cdot U^2 \cdot r^2} = f \left( \frac{\rho \cdot U \cdot r}{\mu} \right) \quad g \left( \frac{r \cdot N}{U} \right)\]

where \((f)\) and \((g)\) are arbitrary functions to be determined in experiments. Note the \((\text{rpm})\) or \((\text{N})\) refers to the rotation in radian per second even though \((\text{rpm})\) refers to revolution per minute. It has to be mentioned that these experiments have to constructed in such way that the initial conditions and the boundary conditions are somehow "eliminated." In practical purposes the thrust is a function of Reynolds number and several other parameters. In this example, a limited information is provided on which only Reynolds number with a additional dimensionless parameter is mentioned above.

Example 9.6

The surface wave is a small disturbance propagating in a liquid surface. Assume that this speed for a certain geometry is a function of the surface tension, \(\sigma\), density, \(\rho\), and the wave length of the disturbance (or frequency of the disturbance). The flow–in to the chamber or the opening of gate is creating a disturbance. The knowledge when this disturbance is important and is detected by with the time it traveled. The time control of this certain process is critical because the chemical kinetics. The calibration of the process was done with satisfactory results. Technician by mistake releases a chemical which reduces the surface tension by half. Estimate the new speed of the disturbance.

Solution 9.6

In the problem the functional analysis was defined as

\[U = f(\sigma, \rho, \lambda)\]

Equation (24) leads to three equations as
\[ \left( \dfrac{M}{L^2} \right)^a \left( \dfrac{M}{t^2} \right)^b \left( L \right)^c \]

\[ \begin{array}{rrl} 
\mbox{Mass}, M & a + b = & 0 \\
\mbox{Length}, L & -2a + c = & 1 \\
\mbox{time}, t & -2b = & -1
\end{array} \]

\[ U = \sqrt{\dfrac{\sigma}{\lambda \rho \pi}} \]

Example 9.7

Eckert number represent the amount of dissipation. Alternative number represents the dissipation, could be constructed as

\[ \text{Diss} = \dfrac{\mu \left( \dfrac{dU}{d\ell} \right)^2}{\frac{\rho U^2}{\frac{\ell}{U}}} = \dfrac{\mu \left( \dfrac{dU}{d\ell} \right)^2 \ell}{\mu \left( \dfrac{dU}{d\ell} \right)^2 \ell} \]

Show that this number is dimensionless. What is the physical interpretation it could have? Flow is achieved steady state for a very long two dimensional channel where the upper surface is moving at speed, \( U_{up} \), and lower is fix. The flow is pure Couette flow i.e. a linear velocity. Developed an expression for dissipation number using the information provided.

Solution 9.7

The nominator and denominator have to have the same units.
The averaged velocity could be a represented (there are better methods or choices) of the energy flowing in the channel. The averaged velocity is \((U/2)\) and the velocity derivative is \((dU/d\ell) = constant = U/\ell\)\. With these value of the Diss number is

\[
Diss = \frac{\mu \left( \frac{U}{\ell} \right)^2 \ell}{\rho \frac{U^3}{8}} = \frac{4\mu}{\rho \ell U}
\]

The results show that Dissipation number is not a function of the velocity. Yet, the energy lost is a function of the velocity square \((E \propto Diss \mu U)\).