10.2: Heat Transfer

The rate of solidification of a metal during casting is dictated by;

- The excess heat in the liquid metal on pouring,
- The amount of heat produced by the solidification of the metal (the latent heat of fusion),
- The rate at which this heat can be dissipated from the metal.

A simple way to predict the way in which a casting will solidify, is using the Biot number, $Bi$, given by:

$$Bi = \frac{h}{K/L}$$

where $h$ is the heat transfer coefficient between the metal and the mould wall, $K/L$ is the thermal conductance of the casting, calculated from $K$, the thermal conductivity of the liquid metal, and $L$, the length of the casting in the direction of the heat flow.

When the Biot number is large, heat is transferred quickly from the metal to the mould, but takes longer to reach the mould wall from the centre of the casting, resulting in a significant temperature gradient in the casting, and only a small temperature difference across the interface.

When the Biot number is small, the transfer of heat to the edge of the casting is faster than the transfer of heat from the metal to the mould, resulting in a large temperature difference across the interface, and only a small temperature gradient within the casting. This is illustrated in the movie below:

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For the situation where $Bi \ll 1$, we can consider the cooling and solidification separately.

Firstly, the liquid metal cools uniformly from its pouring temperature, $T_p$, to its melting temperature, $T_m$. We know that:

\[
q = h \Delta T
\]

and

\[
q = \frac{dT}{dt} C_v L
\]

so that

\[
\frac{dT}{dt} = \frac{h \Delta T}{L C_v}
\]

where $dT / dt$ is the cooling rate, and $C_v$ is the heat capacity of the liquid.

The time, $t_C$, taken for the casting to reach $T_m$ is given by:

\[
t_C = \frac{L C_v}{h} \ln \left| \frac{T_W - T_m}{T_W - T_p} \right|
\]

where $T_W$ is the temperature of the mould wall. To see how this is derived click here.

The solidification stage can be understood by equating the heat transferred across the interface, with the heat generated by the solidification:

\[
q = h \Delta T
\]

\[
q = v \Delta H_F
\]

so that

\[
v = \frac{h \Delta T}{\Delta H_F}
\]

where $v$ is the speed of the solidification front, and $\Delta H_F$ is the latent heat of fusion of the metal.

The time, $t_S$, taken for the metal to solidify once it has reached $T_m$ is given by:

\[
t_S = \frac{\Delta H_F L}{h(T_W - T_m)}
\]

The total time taken for the casting to solidify once the metal has been poured in is:

\[
t = \frac{L}{h} \left( C_v \ln \left| \frac{T_W - T_m}{T_W - T_p} \right| - \frac{\Delta H_F}{(T_W - T_m)} \right)
\]
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