11.4.1: Stagnation State for Ideal Gas Model

It is assumed that the flow is quasi one-dimensional (that is the fluid flows mainly in one dimension). Figure (??) describes a gas flow through state is very useful in simplifying the solution and treatment of the flow. The stagnation state is a theoretical state in which the flow is brought into a complete motionless conditions in isentropic process without other forces (e.g. gravity force). Several properties that can be represented by this theoretical process which include temperature, pressure, and density et cetera and denoted by the subscript "i(0)." First, the stagnation temperature is calculated. The energy conservation can be written as

\[ h + \frac{U^2}{2} = h_0 \]

\[ C_{p} \, T + \frac{U^2}{2} = C_{p} \, T_0 \]

\[ T_0 \] is denoted as the stagnation temperature. Recalling from thermodynamic the relationship for perfect gas \( R = C_{p} - C_{v} \) and denoting \( k \equiv \frac{C_{p}}{C_{v}} \) then the thermodynamics relationship obtains the form

\[ C_{p} = \frac{k \, R}{k - 1} \]

Dividing equation (21) by \( (C_{p} \, T) \) yields

\[ 1 + \frac{U^2}{2 \, C_{p} \, T} = \frac{T_0}{T} \]

Now, substituting \( c^2 = \frac{k \, R \, T}{k} \) or \( T = c^2 / (k \, R) \) equation (23) changes into

\[ C_{p} = \frac{\text{dfrac}{k \, R} \{ k -1 \} \text{iso:eq:tempSimple}}{C_{p} \text{iso:eq:CvRN}} \]
By utilizing the definition of \( k \) by equation (??) and inserting it into equation (??) yields

\[
1 + \frac{(k - 1)}{2}\frac{c^2}{u^2} = \frac{T_0}{T}
\]

It's very useful to convert equation (24) into a dimensionless form and denote Mach number as the ratio of velocity to speed of sound as

\[
M \equiv \frac{u}{c}
\]

into equation (25) reads

Isentropic Temperature relationship

\[
\frac{T_0}{T} = 1 + \frac{k - 1}{2} M^2
\]

Fig. 11.5 Perfect gas flows through a tube.

The usefulness of Mach number and equation (27) can be demonstrated by the following simple example. In this example a gas flows through a tube (see Figure 11.5) of any shape can be expressed as a function of only the stagnation temperature as opposed to the function of the temperatures and velocities. The definition of the stagnation state provides the advantage of compact writing. For example, writing the energy equation for the tube shown in Figure 11.5 can be reduced to

\[
\dot{Q} = C_p \left( T_0 - T_B - T_A \right) \dot{m}
\]

Isentropic Pressure Definition
\[
\frac{P_0}{P} = \frac{T_0}{T}^{k/(k-1)} = \left(1 + \frac{k-1}{2}M^2\right)^{k/(k-1)}
\]

In the same manner the relationship for the density ratio is

\[
\frac{\rho_0}{\rho} = \frac{T_0}{T}^{1/(k-1)} = \left(1 + \frac{k-1}{2}M^2\right)^{1/(k-1)}
\]

New useful definitions are introduced for the case when \(M=1\) and denoted by superscript "∗." The special cases of ratio of the star values to stagnation values are dependent only on the heat ratio as the following:

Star Relationship

\[
\begin{array}{rcccl}
\frac{T^*}{T_0} &=& &\frac{\rho_1}{\rho_2} &=& \left(\frac{P_1}{P_2}\right)^{\frac{1}{k}} \\
\frac{P^*}{P_0} &=& &\left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \\
\frac{\rho^*}{\rho_0} &=& &\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}
\end{array}
\]

Using all the definitions above relationship between the stagnation properties to star speed of sound are

\[
\frac{c^*}{c_0} = \sqrt{k/R} \frac{\sqrt{2T_0}}{\sqrt{k+2}}
\]
Fig. 11.6 The stagnation properties as a function of the Mach number, \( m = 1.4 \).
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